0.1. Cross Product of vectors.

(1) Find the area of the triangle with edges given by vectors

$$\langle 1,1,1\rangle, \langle 2,-1,0\rangle$$

Solution: The area of such a triangle is $|\vec{u}\times\vec{v}|/2$. This gives us

$$\begin{aligned}\langle 1,1,1\rangle\times\langle 2,-1,0\rangle =& \begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 1 & 1 & 1\\ 2 & -1 & 0\end{vmatrix}\\ =&\hat{i}\begin{vmatrix}1 & 1\\ -1 & 0\end{vmatrix} - \hat{j}\begin{pmatrix}1 & 1\\ 2 & 0\end{pmatrix} + \hat{k}\begin{vmatrix}1 & 1\\ 2 & -1\end{vmatrix}\\ =&(1-0)\hat{i} - (0-2)\hat{j} + (-1-2)\hat{k} = \qquad \langle 1,2,-3\rangle\end{aligned}$$

The length of this vector is $\sqrt{1^2+2^2+(-3)^2} = \sqrt{14}$, so the area of the triangle is $\sqrt{14}/2$.

(2) For which value of a is the following cross product the zero vector?

$$\langle 2,-2,3\rangle\times\langle 1,-1,a\rangle$$

Solution: We have that if $\vec{v} = s\vec{u}$, then $\vec{v}\times\vec{u} = 0$. When $a = \frac{3}{2}$, the first vector is a multiple of the second, and so the cross product is zero.

(3) When in general is $\vec{v}\times\vec{u}$ 0?
Solution: We have that if $\vec{v} = s\vec{u}$, then $\vec{v}\times\vec{u} = 0$.

(4) Explain that if $\vec{v},\vec{u}$ satisfies

$$|\vec{u}\times\vec{v}| = \vec{u}\cdot\vec{v} = 0$$

that one of $\vec{v}$ or $\vec{u}$ must be the zero vector.

0.2. **Lines and Planes.**

(1) Find a unit vector perpendicular to the plane $3x + y - z = 2$.

(2) Find a plane P that contains the origin and the line ℓ defined by

$$\begin{aligned} x =& t + 1 \\ y =& -1 \\ z =& -t \end{aligned}$$

Solution: We need to find 3 points on the plane. The points we can use this example wil be given by the origin, and 2 points on the line, $\ell(0) = (1, -1, 0)$ and $\ell(1) = (2, -1, -1)$. Then we have two vectors in this plane, given by $\langle 1, -1, 0\rangle = (1, -1, 0) - (0, 0, 0)$ and similarly $\langle 2, -1, -1\rangle$. The normal to the plane can be found by taking the cross product of these vectors, giving us $\vec{N} = \langle 1, 1, 1\rangle$. So, the equation for the plane is given by

$$(1)x + (1)y + (1)z = c$$

for some constant c that we need to find. By checking this equation on a single point (say, the origin,) we get that $c = 0$. So the equation of the plane is

$$(1)x + (1)y + (1)z = 0$$