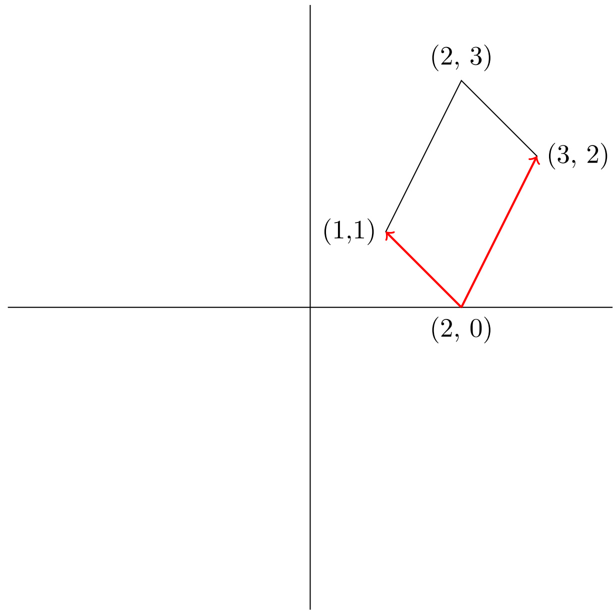
Discussion Problems, August 23

(1) Find the length of both diagonals on a parallelogram with corners

$$(1,1),(2,3),(2,0),(3,2)$$

Solution: Let's first draw a picture of what is happening here:



The sides of the parallelograms are given by the vectors $\vec{v}$ and $\vec{u}$, which are the vectors

$$\vec{v} = \langle 3,2\rangle - \langle 2,0\rangle = \langle 1,2\rangle$$
$$\vec{u} = \langle 1,1\rangle - \langle 2,0\rangle = \langle -1,1\rangle$$

The diagonals of the parallelogram are $\vec{u}+\vec{v}$ and $\vec{v}-\vec{u}$, which have lengths

$$|\vec{u}+\vec{v}| = |\langle 0,3\rangle|$$
$$= \sqrt{0^2+3^2} = 3$$
$$|\vec{v}-\vec{u}| = |\langle 2,1\rangle|$$
$$\sqrt{2^2+1^2} = \sqrt{5}$$

(2) A *Rhombus* is a parallelogram with all 4 edges the same length. Prove that a Rhombus has diagonals which are perpendicular to each other. (Remember, a proof is any explanation of why something is true!)

Solution: Let $\vec{v}$ and $\vec{u}$ be the two sides of the Rhombus. Then the diagonals are $\vec{v}+\vec{u}$ and $\vec{v}-\vec{u}$. The dot product of the diagonals are

$$(\vec{v}+\vec{u})\cdot(\vec{v}-\vec{u}) = \vec{v}\cdot\vec{v} - \vec{v}\cdot\vec{u} + \vec{u}\cdot\vec{v} - \vec{u}\cdot\vec{u}$$
$$= \vec{v}\cdot\vec{v} - \vec{u}\cdot\vec{u} = |\vec{v}|^2 - |\vec{u}|^2 = 0$$

(3) Find the unit vector which is tangent to the circle $x^2 + y^2 = r^2$ at the point $(r\cos\theta, r\sin\theta)$.

Solution: There are several ways to solve this. One way is to use geometry: the line which is tangent to the circle is perpendicular to the radial vector, so we need to take a vector which is unit and perpendicular to $\vec{v} = \langle r\cos\theta, r\sin\theta\rangle$. One way to produce a perpendicular vector in the plane is to switch the x and y components, and put a minus sign in front of one of them. In this case, $\vec{u} = \langle -r\sin\theta, r\cos\theta\rangle$ would be a vector perpendicular to $\vec{v}$. The unit vector would then be

$$\frac{\vec{v}}{|\vec{v}|} = \langle -\sin\theta, \cos\theta\rangle.$$

(4) Discuss with somebody else the philosophical differences between points (x, y, z) and a vector $\langle a, b, c\rangle$. Both of them are geometric objects, so why use two different kinds of notation?