QUIZ 5

NAME:

**Problem 1: True or False**

For each of the following four questions answer " True or false". If true, provide a short explanation. If false, provide a counterexample. No credit will be given for T/F without a justification.

(a) If $f(x,y):\mathbb{R}^2\to\mathbb{R}$ is continuous at the point $(0,0)$, and $f_x(0,0)$ is defined, then $f_y(0,0)$ is defined as well.

**Solution:**False. Take for instance

$$f(x,y)=\begin{cases} |y| & x=0 \\ 0 & \text{otherwise}\end{cases}$$

(b) If $f(x,y):\mathbb{R}^2\to\mathbb{R}$ is continuous, $f_x(0,0)$ and $f_y(0,0)$ are defined at the origin, then $f$ is differentiable at the origin.

**Solution:**False. Take for instance

$$f(x,y)=\begin{cases} 0 & y,x=0 \\ \frac{xy}{x^2+y^2} & \text{otherwise}\end{cases}$$

(c) If $f(x,y):\mathbb{R}^2\to\mathbb{R}$ is a function which is differentiable at $(0,0)$, then it continuous everywhere.

**Solution:**False. Take for instance

$$f(x,y)=\begin{cases} 42 & y,x=1 \\ 0 & \text{otherwise}\end{cases}$$

(d) If $f(x,y):\mathbb{R}^2\to\mathbb{R}$ is a function with $|f(x,y)|\le x^2+y^2$, then $f$ is differentiable at the origin.

**Solution:**False. Take for instance

$$f(x,y)=\sqrt{x^2+y^2}$$

**Problem 2: Chain Rule**

Let $x(s,t)$ and $y(s,t)$ be two functions.

Let $z(s,t)=x^2+x+y^2+\cos(y)$. Suppose we know that $\frac{\partial x}{\partial s}=\cosh(s+t)$, $\frac{\partial y}{\partial s}=\ln(1+s+\sin(t^3))$. Furthermore, suppose $x(0,0)=y(0,0)=0$. Compute what $\frac{\partial z}{\partial s}$ is when $s=t=0$.

**Solution:**Applying the chain rule we have that

$$\begin{aligned}\frac{\partial z}{\partial s} =&\frac{\partial z}{\partial x}\frac{\partial x}{\partial s}+\frac{\partial z}{\partial y}\frac{\partial y}{\partial s}\\ =&(2x+1)\cosh(s+t)+(2y-\sin(y))(\text{Some other term})\end{aligned}$$

Letting $x=y=s=t=0$

$$\begin{aligned}=&(0+1)\cosh(0+0)+(0-0)(\text{Some other term})\\ =&1\end{aligned}$$

**Problem 3: Tangent Planes** Let $f(x, y) = \sin x \cos y$. Find the tangent plane to this surface at the origin. First you must find the $x$ and $y$ partial derivatives. $f_x = \cos x \cos y$, and $f_y = -\sin x \sin y$. At the origin, we have that

$$\begin{aligned} f_x(0,0) =& 1 \\ f_y(0,0) =& 1 \end{aligned}$$

The formula for the tangent plane is given by

$$\begin{aligned} z(s,t) =& s f_x(x_0, y_0) + t f_y(x_0, y_0) + f(x_0, y_0) \\ =& s \end{aligned}$$

**Interesting Puzzle, Will not be graded.** There is a pirate ship commanded by a just, communal, and mathematically minded pirate captain. Though she is a fearsome pirate, she lives by the pirate code when distributing booty to her mates. The pirate code states:

> The Pirate captain shall come up with a plan to distribute booty. That plan then goes to the crew for a vote. If the vote is in favor or a tie, the plan passes and booty is distributed. The pirates then go off to plunder more treasure. If the vote fails to pass, then the captain shall be forced to walk the plank and jump into shark infested waters. The first mate shall become the captain, the second mate shall be promoted to first mate, and so on ( the $(n+1)$-mate will be promoted to $n$-mate.) The new captain will come up with a plan, and the process above repeated until booty has been distributed.

The pirate captain knows her 50 crew members well. In particular, she knows that all pirates vote on the following preferences:

- Pirates look out for their own neck before thinking about plunder— that is, no pirate will propose a plan to distribute booty that will cause them to walk the plank if they can afford to.
- Pirates prefer more treasure to less treasure, and don't really care about friendships. They don't make promises beyond following the pirate code.

One day, the pirates come across 100 gold coins to distribute. What is the final distribution of gold coins among the pirates?