QUIZ 4

NAME:

(a) Find a parameterization $\vec{l}(s)$ for the tangent line at the point $\vec{f}(0)$ for the vector valued function

$$\vec{f}(t) = \langle \sin t, \cos t, t \rangle$$

Solution: We have that the velocity vector of a parameterized curve is given by taking derivatives in each of its components, giving us

$$\vec{f}'(t) = \langle \cos t, -\sin t, 1 \rangle$$

At $t = 0$, this is the vector $\langle 1, 0, 1 \rangle$. The line that travels in this direction that starts at the point $\vec{f}(0)$ is given by

$$\vec{l}(s) = \langle t + 0, 1, t + 0 \rangle$$

(b) Find the length of the curve between time 0 and π.

Solution: The length of the curve is given by $\int_0^2 \pi \frac{ds}{dt} dt$, where

$$\frac{ds}{dt} = \sqrt{\frac{dx}{dt} + \frac{dy}{dt} + \frac{dz}{dt}} = \sqrt{2}$$

Therefore the length of the curve is $\pi\sqrt{2}$.

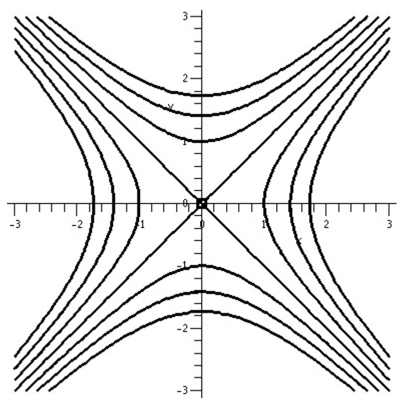
Problem 2: The Saddle:

The saddle is given by the function

$$z = x^2 - y^2$$

Draw a contour plot of the saddle, for integer values of z between -3 and 3.

Solution: Here is such a drawing. The level curves are not for the integer z values between -3 and 3, but the shape is generally correct.



Problem 3: Continuity
Consider the function

$$z = \begin{cases} \frac{xy}{x^2+y^2} & (x,y) \neq (0,0) \\ 0 & (x,y) = (0,0) \end{cases}$$

Determine range of the function, and the domain on which the function is continuous.

Solution: Let's start by computing the domain of continuity. We know that the function must be continuous except possibly where the denominator $x^2 + y^2 = 0$. Here, it is not continuous– observe that

$$\lim_{n\to\infty} f(1/n, 1/n) = 1/2$$
$$\lim_{n\to\infty} f(0, 1/n) = 0$$

so the function is not continuous at the origin.
For the range, we have that $(x-y)^2 = x^2 - 2xy + y^2 \geq 0$, which tells us that $x^2 + y^2 \geq 2xy$. This tells us that the value of the function can never be bigger than 1/2. It indeed achieves this value along the line $x = y$. Likewise, we see that the minimum value of this function is $\frac{-1}{2}$, which it achieves along the line $x = -y$.

Interesting Puzzle, Will not be graded. There is a prison run by a strange and mathematically inclined warden. He gathers 20 prisoners in a room, and gives them colorful hats. No prisoner can see their own hat, but can see all the other hats. The warden then says "At least one of you is wearing a red hat. I will ask all of you together to raise your hand if you are wearing a red hat. If one of you raises your hand incorrectly, I will take away all of your meals for the week. If everybody who is wearing a red hat correctly identifies themselves, I will let you all go free. There is no penalty for not raising your hand. Also, I will continue to ask the group 'Are you wearing a red hat' until either somebody is wrong, or all the red hats have been identified."
So he begins: "Are you wearing a red hat? "
As nobody knows the color of their own hat, nobody raises their hands. And he asks the group again, "Are you wearing a red hat? " Is there a way for a group of rational prisoners to save themselves ?