Here are a few problems related to things we have looked at so far.

Problem 1

(a) What shape is each one of these quadratics? What is the shape of their intersection? It may help you to draw out the quadratics.

$$xy + z^2 = 1$$
$$x^2 + y^2 + z^2 = 2$$

(b) The intersection of the two quadratics has two components. One of the components contains the point (1,0,1). Find a plane that contains this component.

Problem 2

Look at the polar curve

$$r(\theta) = \sqrt{2}\csc\theta$$

(a) Find the length of the curve where $\pi/4 \leq \theta \leq 3\pi/4$

(b) Find the area bounded by the x axis and the curve, where $\pi/4 \leq \theta \leq 3\pi/4$.

Problem 3

The slipped cycloid is given by the equations

$$\begin{aligned} x(t) &= \sin(2t) + t \\ y(t) &= \cos(2t) \end{aligned}$$

Find the length of the curve as t ranges between 0 and 2π.