QUIZ 1

NAME:

Problem 1: A Flower:
This is a polar function for a flower:

$$r(\theta) = \sin^2(3\theta)$$

where θ takes a value between 0 and 2π.

- How many petals does this flower have?

  Solution: $\sin^2$ always takes a non-negative value, and between 0 and 2π it takes a value of zero 6 times, so it has 6 petals.
- What is the area enclosed by the flower?

  Solution: We first set up the integral for the area of the curve:

$$\begin{aligned} \text{Area} &= \int_0^{2\pi} \frac{(r(\theta))^2}{2} d\theta \\ &= \frac{1}{2} \int_0^{2\pi} \sin^4(3\theta) d\theta \end{aligned}$$

  Using the power reduction formula twice

$$\begin{aligned} &= \frac{1}{2} \int_0^{2\pi} \frac{-4\cos(6\theta) + \cos(12\theta) + 3)}{8} d\theta \\ &= 3\pi/8 \end{aligned}$$

Problem 2: Polar to Parametric:
An ellipse (with axis of length a is given in polar coordinates by the function

$$r(\theta) = \frac{ab}{\sqrt{(b\cos\theta)^2 + (a\sin\theta)^2}}$$

- Write down an integral which computes the length of the above curve. Do *not* evaluate the integral.
  Solution: The arclength formula for a curve in polar coordinates is given by

$$\text{Arclength} = \int_0^{2\pi} \sqrt{(r(\theta)^2 + \left(\frac{dr}{d\theta}\right)^2} d\theta$$

  Making appropriate substitutions

$$= \int_0^{2\pi} \sqrt{\left(\frac{ab}{\sqrt{(b\cos\theta)^2 + (a\sin\theta)^2}}\right)^2 + \left(\frac{ab((-1/2)(b\cos\theta)^2 + (a\sin\theta)^2)^{\frac{-1/2}{2}} b(\cos\theta)(-\sin(\theta) + 2a\sin\theta\cos\theta)}{(b\cos\theta)^2 + (a\sin\theta)^2}\right)^2} d\theta$$

You could leave your solution like this.

- Rewrite this graph as a parametric curve in (x, y) coordinates.
  Solution: Knowing that $x = r\cos\theta$, and $y = r\sin\theta$, you get that
  $$x(t) = a\cos(\theta)$$
  $$y(t) = b\sin(\theta)$$

Problem 3:
Show (using identities or formulas for cross product and dot product) that for any vectors $\vec{a}, \vec{b}$ we have
$$\vec{a} \cdot (\vec{a} \times \vec{b}) = 0$$

Solution: Write out vector $\vec{a}$ as $a_1 i + a_2 j + a_3 k$, and vector $\vec{b}$ as $b_1 i + b_2 j + b_3 k$. Then writing out
$$\begin{aligned}\vec{a} \cdot (\vec{a} \times \vec{b}) =& (a_1 i + a_2 j + a_3 k) \cdot ((a_1 i + a_2 j + a_3 k) \times (b_1 i + b_2 j + b_3 k)) \\ =& (a_1 i + a_2 j + a_3 k) \cdot ((a_2 b_3 - b_2 a_3) i - (a_1 b_3 - b_1 a_3) j + (a_1 b_2 - b_2 a_1) k) \\ =& (a_1 a_2 b_3 - a_1 b_2 a_3) - (a_2 a_1 b_3 - a_2 b_1 a_3) + (a_3 a_1 b_2 - a_3 b_2 a_1)\end{aligned}$$

The appropriate terms all match up and cancel

Interesting Puzzle, Will not be graded. There is a prison run by a strange and mathematically inclined warden. The warden is throwing a party for guests, and has 1000 barrels of delicious fruit juice prepared. Unfortunately last night one of the prisoners poisoned one of the barrels.
The warden decides to use the prisoners to determine which barrels are poisone, by forcing them to drink the juice. As the poison takes 12 hours to take effect, he or she will only have time to run 1 such test. One example of a test would be to have 1000 prisoners each sip from a different barrel. This test has the downside that the warden has to serve juice to 1000 prisoners. Can you determine a way to check which barrel is poisoned in such a way that fewer than 20 prisoners will wind up drinking juice?