QUIZ 1

NAME:

**Problem 1: Graphing Parametrics:**
The golden ratio spiral is given by the equation

$$\begin{aligned} x(t) =& e^{\phi t}\cos(t) \\ y(t) =& e^{\phi t}\sin(t) \end{aligned}$$

where $\phi = \frac{1+\sqrt{5}}{2}$ is the golden ratio.

- Using any method you wish, draw a graph of this for parameter $0 \leq t \leq 6\pi$. **Solution:**You should get a spiral, rotating counterclockwise, starting at the point $(1, 0)$ and going around 2 times.
- For what value of $0 \leq t \leq 6\pi$ does this graph take on a maximal value? Taking a derivative of the $y$ coordinate and setting it equal to zero gives us

  $$\phi e^{\phi t}\sin(t) + e^{\phi t}\cos(t) = 0$$

  Pulling out a $\phi t$ we have that

  $$\phi \sin t + \cos t = 0$$

  Solving for $t$ gives us

  $$t = \cot^{-1}(\phi)$$

**Problem 2: Derivatives of Parametrics:**
The regular spiral is given by the equation

$$\begin{aligned} x(t) =& t\cos(t) \\ y(t) =& t\sin(t) \end{aligned}$$

Find a point on the spiral that has a slope of $\pi$.
**Solution:**Taking a derivative gives us that

$$\begin{aligned} \frac{dx}{dt} =& -t\sin(t) + \cos(t) \\ \frac{dy}{dt} =& t\cos(t) + \sin(t) \end{aligned}$$

Giving us $\frac{dy}{dx}|_{t=\pi} = \frac{\pi\cos(\pi)}{\cos(\pi)} = \pi$

**Problem 3: Integration of Parametrics:**
Using the fact that

$$x(t) = \sin(t)$$
$$y(t) = \cos(t)$$

parameterizes the unit circle, prove that the area of the unit circle is $\pi$.
**Solution:**We will find the area to the top half of the circle. Take $\int x(t)\frac{dy}{dt}dt$ gives us

$$\int_0^\pi \sin(t)(-\sin(t))dt$$
$$= -\int_0^\pi \frac{1-\cos 2\theta}{2}$$
$$= \frac{\theta - \sin 2\theta}{2}\Big|_0^\pi$$
$$= \frac{\pi}{2}$$

Doubling this gives an earea of $\pi$.

**Interesting Puzzle, Will not be graded.** There is a prison run by a strange and mathematically inclined warden. The Warden assembles all 100 of his prisoners in a line. He then puts a red hat or a blue hat on each of the 100 prisoners.
No prisoner can see the color of the hat on his or her head, but can see the hats on all of the prisoners in front of them in the line. The Warden starts at the end of the line, and barks, "What color is your hat?!", to which the prisoner is only allowed to reply "Red!" or "Blue!"
The other prisoners can hear what the prisoners behind them say.
If a prisoner correctly identifies the color of their own hat, then they are allowed to go free, otherwise, they are condemned to a lifetime of doing pushups.

Can you come up with a strategy for the prisoners that saves

- At least 1 prisoner?
- At least 50 prisoners?
- At least 51 prisoners?
- Almost all of the prisoners?