

QUIZ 1

NAME:

Review of Precalculus.

- (a) What is $\log(e^2) - \log(e^3)$.

Solution:

$$\begin{aligned}\log(e^2) - \log(e^3) &= 2\log(e) - 3\log(e) \\ &= -1\end{aligned}$$

- (b) Express $\log(ab^c)$ in terms of $\log(a)$ and $\log(b)$.

Solution: $\log(ab^c) = \log(a) + \log(b^c) = \log(a) + c\log(b)$

- (c) $\cos(2\pi/3)$ is what value?

Solution: $-\frac{1}{2}$

Review of Differentiation.

 Compute the following derivatives:

- (a) What is the derivative of $\log(x^2)$?

Solution: You can solve this either by chain rule, or by using log rules. If you use log rules, you have

$$\begin{aligned}\frac{d}{dx} \log(x^2) &= \frac{d}{dx} 2\log(x) \\ &= \frac{2}{x}\end{aligned}$$

- (b) What is the maximal value of the function $-x^4 + 4x$?

Solution: You'll want to figure out where the derivative is zero, and evaluate the function there. The derivative of the function is

$$f'(x) = -4x^3 + 4x$$

which takes on a zero at when $x = 1$. You can check by your favorite method (like second derivative test) that this is a maximum. Therefore, the maximal value of this function is $f(1) = 3$.

- (c) What is the slope of the line going through the point $(\frac{1}{2}, \frac{\sqrt{3}}{2})$ tangent to the curve

$$x^2 + y^2 = 1$$

Solution: This is a typical implicit differentiation problem. We have that

$$2xdx + 2ydy = 0$$

“Solving” for dy/dx gives us that

$$\frac{dy}{dx} = \frac{-x}{y}$$

which means that the slope of the line will be $\frac{1}{\sqrt{3}}$.

Review of Integration. Compute the following integrals:

(a) Find the integral of

$$\int x \sin(x) dx$$

Solution: This is an integration by parts problem. Setting $u = x$, $dv = \sin(x)$ we get

$$\begin{aligned} \int x \sin(x) dx &= -x \cos(x) - \int -\cos(x) dx \\ &= -x \cos(x) + \sin(x) + C \end{aligned}$$

(b) Find the integral of

$$\int \frac{x^2}{(x+1)} dx$$

Solution: This is a polynomial division problem. Using polynomial long division

$$\begin{aligned} \int \frac{x^2}{(x+1)} dx &= \int x - 1 + \frac{1}{x+1} \\ &= \frac{x^2}{2} - x + \log(x+1) + C \end{aligned}$$

Interesting Puzzle, Will not be graded. There is a prison run by a strange and mathematically inclined warden. The Warden assembles all 100 of his prisoners in a room, and gives them the following puzzle:

“Every day, a Guard will bring one of you (chosen at random) into a room. There will be a lightbulb in the room, and it will be on or off. I will give you the option of turning the lightbulb on or off. The Guard will then ask you if all your fellow prisoners have been in the room yet.

- If you stay silent, things will go on as normal, and tomorrow another random prisoner will be brought into the room.
- If you say yes, and you are incorrect, I will sentence you all to a life in prison without parole.
- If you say yes, and are correct, then you will all go free.”

“Now, I promise you that I will not change the state of lightbulb. Furthermore, to be fair to you I will now give you a few minutes to discuss a strategy before putting you all back in your cells, where you will remain in total isolation until you solve the puzzle.”

How can the prisoners best insure that their incarceration only lasts for a finite time?