Problem: Find the surface area of the shape generated by rotating the curve $y^2 = x^3$ around the y axis, where x takes on values between 0 and 1.

Solution: We know that the surface area of a rotation around the y axis is given by

$$\int_a^b 2\pi x ds$$

We have that $ds = \sqrt{1-(f'(x))^2}\,dx$. Computing $f'(x) = \frac{3}{2}x^{1/2}$,

$$\begin{aligned}\int_0^1 2\pi x\sqrt{1+(f'(x))^2}\,dx =& \int_a^b 2\pi x\sqrt{1-(\frac{3}{2}x^{1/2})^2}\,dx \\ =&2\pi\int_0^1 x\sqrt{1+\frac{9}{4}x}\,dx\end{aligned}$$

Substituting $u = 1 + \frac{9}{4}x$, and $du = \frac{9}{4}dx$

$$\begin{aligned}=&2\pi\int_{u(0)}^{u(1)} \frac{4}{9}(u-1)\sqrt{u}\,\frac{4}{9}du \\ =&2\pi\int_{u(0)}^{u(1)} \frac{16}{81}\left(u^{3/2}-u^{1/2}\right)du \\ =&2\pi\frac{16}{81}\left(\frac{2}{5}u^{5/2}-\frac{2}{3}u^{3/2}\right)\Bigg|_{u(0)}^{u(1)}\end{aligned}$$

If you got this far in the problem, you got full credit. Substituting back in $u(0) = 1$, and $u(1) = 1 + \frac{9}{4}$

$$=2\pi\frac{64+247\sqrt{13}}{1215}$$