1. Problems to be presented on 10-11

If you are interested in doing a problem, but would like some help, email me for hints.

- First problem for presentation: Suppose that a_n is a decreasing sequence greater than 0, that is
$$a_1 \geq a_2 \geq \ldots \geq 0$$
Show that if $\sum_{n=1}^{\infty}$ converges, then the series
$$\sum_{k=0}^{\infty} 2^k a_{2^k} = a_1 + 2a_2 + 4a_4 + 8a_8 + \ldots$$
converges as well.
- Second Problem for Presentation: Suppose that a_n is a decreasing sequence greater than 0, that is
$$a_1 \geq a_2 \geq \ldots \geq 0$$
Show that if $\sum_{k=0}^{\infty} 2^k a_{2^k} = a_1 + 2a_2 + 4a_4 + 8a_8 + \ldots$ converges, then
$$\sum_{n=1}^{\infty} a_n$$
converges as well.
- Third problem for presentation: Suppose that $\sum_{n=1}^{\infty} a_n = 0$. Can you rearrange the terms of the sum so that it converges instead to 1? (You may want to look up the greedy algorithem)