QUIZ 5

NAME:

Problem 1. If the following sequence or series converges, write its limit, otherwise write diverges. You do not need to prove convergence or divergence. Each one is worth 2 points. Each wrong answer is worth −1.

(a) $\lim_{n\to\infty} \frac{4n^2+4}{(n+1)(2n+1)}$

2

(b) $\lim_{n\to\infty} \frac{e^n}{n^n}$

0

(c) $\lim_{n\to\infty} \log(2n^3) - \log(2n^3+5)$

0

(d) $\lim_{n\to\infty} \frac{\log n}{\sin 1/n}$

Diverges

(e) $\lim_{n\to\infty} \sin(\frac{1}{n})$

0

Problem 2. For what values of r does the following series converge? Show your work.

$$\sum_{n=1}^{\infty} nr^n$$

Solution: This function is monotonic decreasing, and "nice", so you might use the integral comparison test.

$$\begin{aligned}\int_0^{\infty} xr^x\,dx &= \lim_{t\to\infty}\int_0^t xr^x\,dx\\ &= x\frac{1}{\ln(r)}r^x + \frac{1}{(\ln(r))^2}r^x\Big|_0^t\end{aligned}$$

This converges only when r^x converges to 0 as $x \to \infty$, which only happens when r is between −1 and 1.

Problem 3. The Fibonacci sequence is defined by the recurrence relation

$$f_n = f_{n-1} + f_{n-2}.$$

Starting with $f_0 = 0, f_1 = 1$. So the first few terms are $0, 1, 1, 2, 3, 5, 8, 13, \ldots$ Define the sequence $a_n = \frac{f_n}{f_{n-1}}$ whenever $n \geq 1$, and 0 otherwise. So the first few terms of the sequence are

$$0, 0, \frac{1}{1}, \frac{2}{1}, \frac{3}{2}, \frac{5}{3}, \ldots$$

Show that the sequence a_n converges, and find what it converges to. Be sure to show your work!
Solution: We have the equalities

$$\begin{aligned} a_n =& \frac{f_n}{f_{n-1}} \\ =& \frac{f_{n-1} + f_{n-2}}{f_{n-1}} \\ =& 1 + \frac{f_{n-2}}{f_{n-1}} \\ =& 1 + \frac{1}{a_{n-1}} \end{aligned}$$

If the sequence a_n converges, it converges to some value L. Pluggin in that value for a_n we have that

$$L = 1 + \frac{1}{L}$$

Solving for L yields $L = \frac{1 \pm \sqrt{5}}{2}$. Since L must be positive (as the quotient of two fibonacci numbers is positive) we have $L = \frac{1 = \sqrt{5}}{2}$.