1. Astroid:

Last week on the homework, there was a problem that asked:
Calculate the length of the astroid of $x^{\frac{2}{3}} + y^{\frac{2}{3}} = 2$.
Solution: We will find the arclength in the first quadrant, and then by symmetry we can take our solution and multiply by four. Using implicit differentiation,

$$\frac{2}{3}x^{-\frac{1}{3}}dx + \frac{2}{3}y^{-\frac{1}{3}}dy = 0$$

Solving for y' and squaring it (and substituting so the solution is in terms of x we have,

$$(y')^2 + 1 = \frac{2}{x^{\frac{2}{3}}}$$

Therefore, the arclength (of one quarter) is given by

$$\begin{aligned} s = &\int_0^{2^{\frac{3}{2}}} \sqrt{((y')^2 + 1)}dx \\ = &\int_0^{2^{\frac{3}{2}}} \frac{\sqrt{2}}{x^{\frac{1}{3}}}dx \\ = &\sqrt{2}\frac{3x^{\frac{2}{3}}}{2}\Bigg|_0^{2^{\frac{3}{2}}} \\ = &\frac{3}{2}\sqrt{2} \times 2 = \frac{3}{2}2^{\frac{3}{2}} \end{aligned}$$

Therefore, the total arclength is

$$6 \cdot 2^{\frac{3}{2}}$$