Quiz 1

Name:

Problem 1. Find the integral of

$$\int x \sin(x)\, dx$$

Solution: We do integration by parts, by letting $u = x$ and $dv = \sin x$. This gives us

$$-x\cos x - \int (-\cos x)\, dx$$

which is equal to

$$-x\cos x + \sin x + C$$

Problem 2. Evaluate

$$\int_0^{\pi} \sin^3(x)\cos^2(x)\, dx$$

Solution: Using the rule $\sin^2(x) = 1 - \cos^2(x)$, we get

$$\int_0^{\pi} \sin x(1 - \cos^2(x))\cos^2(x)\, dx$$

Doing the u substitution of $u = \cos x$, and $du = -\sin x$ we have

$$\begin{aligned}
\int \sin x(1 - \cos^2(x))\cos^2(x)\, dx &= \int_{-1}^{1} -(1 - u^2)u^2\, du \\
&= -\int_{-1}^{1} (u^2 - u^4)\, dx \\
&= -\frac{u^3}{3} + \frac{u^5}{5}\Big|_{-1}^{1} \\
&= \frac{4}{15}
\end{aligned}$$

Problem 3. Find the integral of

$$\int x \tan^{-1}(x)\, d\, x$$

Solution: We first integrate by parts, and then use trig substitution. Let $u = \tan^{-1}x$, and $d\, v = x$. Then integrating by parts gives us

$$\int x \tan^{-1}(x)\, d\, x = \frac{1}{2}x^2\tan^{-1}(x) - \int \frac{x^2}{1+x^2} d\, x$$

⟨intertext | ⟩$M\ a\ k\ i\ n\ g\ a\ s\ u\ b\ s\ t\ i\ t\ u\ t\ i\ o\ n\ o\ f x =$ $\tan(\theta)$ on the right hand integral and $d\, x = \sec^2(\theta)\, d\theta\} =$

$$\frac{1}{2}x^2\tan^{-1}(x) - \frac{1}{2}\int \frac{\tan^2(\theta)}{1+\tan^2(\theta)}\mathrm{dx}$$

$$= \frac{1}{2}\left(x^2\tan^{-1}(x) - \int \frac{\tan^2(\theta)}{\sec^2(\theta)}\mathrm{dx}\right)$$

$$= \frac{1}{2}\left(x^2\tan^{-1}(x) - \int \sin^2(\theta)\,\mathrm{dx}\right)$$

$$= \frac{1}{2}\left(x^2\tan^{-1}(x) - \int \sin^2(\theta)\sec^2(\theta)\, d\theta\right)$$

$$= \frac{1}{2}\left(x^2\tan^{-1}(x) - \int \tan^2(\theta)\, d\theta\right)$$

$$= \frac{1}{2}\left(x^2\tan^{-1}(x) - \int 1 - \sec^2(\theta) d\theta\right)$$

$$= \frac{1}{2}\left(x^2\tan^{-1}(x) - \theta + \tan\theta\right) + C$$

$$= \frac{1}{2}\left(x^2\tan^{-1}(x) - \tan^{-1}(x) + x\right) + C$$