1. Additional Review Problems!

(1) Using u substitution, find the integral of $\tan x$

Solution:

$$\int \tan x dx = \int \frac{\sin x}{\cos x} dx$$

Set $u = \cos x$, so that $du = -\sin x dx$

$$\begin{aligned} &= \int \frac{-du}{u} \\ &= -\ln|u| + C \\ &= -\ln|\cos x| + C \end{aligned}$$

(2)

$$\lim_{n\to\infty}\left(\frac{1}{\sqrt{n}\sqrt{n+1}} + \frac{1}{\sqrt{n}\sqrt{n+2}} + \dots \frac{1}{\sqrt{n}\sqrt{n+n}}\right)$$

Think the fundamental theorem of calculus!

Solution: We are going to massage the limit to make it a little better to look at. I'm going to use summation sign here.

$$\lim_{n\to\infty}\sum_{k=1}^{n}\frac{1}{\sqrt{n}\sqrt{n+k}} = \lim_{n\to\infty}\sum_{k=1}^{n}\frac{1}{n}\cdot\frac{1}{\sqrt{1+k/n}}$$

The right side is a Riemann sum— this is easier to see by setting $f(x) = \frac{1}{\sqrt{x}}$

$$= \lim_{n\to\infty}\sum_{k=1}^{n}\frac{f(1+k/n)}{n}$$

Using the definition of the integral

$$\begin{aligned} &= \int_1^2 f(x)dx \\ &= \int_1^2 \frac{1}{\sqrt{x}} dx \\ &= 2x^{1/2}\Big|_{x=1}^{2} \\ &= 2(\sqrt{2}-1) \end{aligned}$$

(3) Compute the following limit

$$\lim_{x\to 4}\frac{1}{x-4}\int_4^x \frac{e^t}{t} dt$$

Solution: There were several good solutions to this one. (Thanks, Vincent and Taylor.) Here are a few:

(a) One way to think of $\frac{1}{x-4}\int_4^x \frac{e^t}{t}dt$ is the average value of $\frac{e^t}{t}$ between 4 and x. As the function $\frac{e^t}{t}$ is continuous at 4, the average value of the function on the interval $[4, x]$ should tend to the value of the function at 4 as x goes to 4. So the answer is $e^4/4$

(b) You can also use L'Hopital rule for this one. Rewriting at

$$\lim_{x\to 4} \frac{\int_4^x \frac{e^t}{t} dt}{x-4}$$

The top of this fraction and bottom of this fraction are both differentiable, and go to 0 as x tends to 4. We may use L'Hopital and get

$$\lim_{x\to 4} \frac{\frac{d}{dx}\left(\int_4^x \frac{e^t}{t} dt\right)}{1}$$

By the fundamental theorem of calculus, this is $e^4/4$.

(c) Suppose that $F(t)$ is the anti-derivative of e^t/t. Then we can use the fundamental theorem of calculus and get

$$\lim_{x\to 4} \frac{1}{x-4} \int_4^x \frac{e^t}{t} dt = \lim_{x\to 4} \frac{F(x)-F(4)}{x-4}$$

The right hand side is the definition of the derivative of F at 4

$$=F'(4)$$
$$=e^4/4$$

(4) Japanese Napkin Ring Theorem.

Solution: Wikipedia has a good write up of this problem, so I will leave it to them. Google : "Japanese Napkin Ring Theorem" and look it up!

If you want additional problems to review, try these two:

(5) Assume a snowball melts so that its volume decreases at a rate proportional to its surface area. If it takes 3 hours for the snowball to decrease to half its original volume, how much longer will it take for the snowball to melt completely

(6) Dart Board Probability Problem: Suppose that I am throwing darts at a dart board of radius 1. The score that I get when I hit the board is equal to $1 - x$, where x is the distance from the center of the board to where my dart landed. So if I make a bullseye, I get 1 point. If I hit the rim, I get zero points. What is my average score?