One of the limits that we computed last week in section was

$$\lim_{h \to 0} (1-2h)^{\frac{1}{h}}$$

We looked at two different ways to compute this limit in class.

Using Logs

In class, we had this explanation:

$$\begin{aligned}
\lim_{h \to 0} (1-2h)^{\frac{1}{h}} &= \exp(\ln(\lim_{h \to 0} (1-2h)^{\frac{1}{h}})) && (1)\\
&= \exp(\lim_{h \to 0} (\ln((1-2h)^{\frac{1}{h}})) && (2)\\
&= \exp\left(\lim_{h \to 0} \frac{\ln(1-2h)}{h}\right) && (3)\\
&= \exp\left(\lim_{h \to 0} \frac{\frac{-2}{1-2h}}{1}\right) && (4)\\
&= \exp(-2) && (5)\\
&= e^{-2} && (6)
\end{aligned}$$

There are two tricky steps here. Between lines (3) and (4), we used LH Rule. The other tricky spot is between (1) and (2), where we need to justify why the natural log of the limit is the same as the limit of the natural log. This is because the natural log function is continuous where the limit might be converging. Remember, if we have that f is a continuous function, then

$$\lim_{x \to x_0} f(x) = f(\lim_{x \to x_0} x)$$

Using the definition of e^x

Another way to do this problem is to remember the definition of the exponentiation function. There is a way to define the number e as

$$e = \lim_{n \to \infty} \left(1 + \frac{1}{n}\right)^n$$

Similarly, there is a definition of the exponential function as

$$e^x = \lim_{n \to \infty} \left(1 + \frac{x}{n}\right)^n$$

If we let $x = -2$, and $h = \frac{1}{n}$, then by substitution we have

$$e^{-2} = \lim_{h \to 0} (1-2h)^{\frac{1}{h}}$$