# MATH N55 HOMEWORK 7
# DUE TUESDAY, JULY 23RD

Do the following problems in Rosen.

**Section 6.2:** 12, 18
**Section 6.3:** 23, 43
**Section 6.4:** 26, 28, 32
**Section 6.5:** 9, 20, 25

## Challenge (Not to be handed in)

Let $F(n,k)$ be the number of surjections $f : \{1,\ldots,n\} \to \{1,\ldots,k\}$. We showed in class that this is $k!\cdot S(n,k)$. This exercise will develop a different way to compute it.

(1) Let $n, m$ be fixed natural numbers. Give a *combinatorial proof* of the following identity:

$$\sum_{k=0}^{m} \binom{m}{k} F(n,k) 2^{m-k} = \sum_{j=0}^{m} \binom{m}{j} j^n$$

(2) Adjust this argument to show that

$$F(n,m) = \sum_{j=0}^{m} (-1)^{m-j} \binom{m}{j} j^n.$$

*Possible hint:* The $2^{m-k}$ from before probably counted subsets of some set with $m-k$ elements. What if instead you counted the difference between even and odd subsets?