

MATH 1B WEEK 8, THURSDAY

Exercise 1. Find a power series for $\frac{x}{x+2}$, around $x = 0$. For what values of x is it valid?

Exercise 2. Find a power series expansion for $\ln(1+x)$, valid for $|x| < 1$. (*Hint:* First show that $\ln(1+x) = \int_0^x \frac{1}{t+1} dt$.)

Exercise 3. The previous exercise gives a series for $\ln(1+x)$ on the interval $(0, 2)$, but it turns out that the same series works on $(0, 2]^1$. Using this fact, compute $1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \frac{1}{5} - \dots$.

¹This is a consequence of Abel's Theorem on power series, which we will not cover

Exercise 4. Write down a power $f(x) = \sum_{n=0}^{\infty} a_n x^n$ with the following properties:

- (1) $f'(x) = f(x)$ for all x .
- (2) $f(0) = 1$.

We won't prove it today, but it this property completely determines f . In particular, if another function has this property, it *is* this series.