Week 9 Worksheet

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1. Decide if each of the following is true or false. Justify your answer.

   In what follows, f is a function, and a, b, c are constants.

   (a) If $f'(x) > 0$ on an interval, then f is positive on that interval.
   (b) If $f'(c)$ exists, then $f''(c)$ exists.
   (c) If $f'(c) = 0$ and $f''(c) = 0$, then c is neither a min nor max of f.
   (d) If f is an odd function whose domain contains 0, then $f(0) = 0$.
   (e) It's possible for a function f defined on (a, b) to not have any relative extrema.

2. Find the locations and values of all relative extrema. Unless otherwise stated, assume the domain of each function is the largest set on which the given formula makes sense.

   (a) $f(x) = -x^2 + 4x - 8$.
   (b) $g(x) = \frac{xe^x}{x-1}$.
   (c) $h(x) = \sqrt[3]{x-2} + 7x$.
   (d) $k(x) = 10 - x^2$, defined on $(-5, 6]$.

3. Sketch graphs of the following functions:

   (a) $\ell(x) = x^2 e^{2x}$
   (b) $m(x) = \frac{-4x}{1+x}$

4. In probability, one of the most commonly occuring distributions is the normal distribution, or "bell curve." It depends on two constants: the *mean* μ, and the *standard deviation* σ. The formula for the distibution is given by

$$f(x) = \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}$$

This function has a single local maximum. Where is it, and what is the value of f there? Where are the points of inflection?