Week 4 Worksheet

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1. On what set is the function $g(x) = \dfrac{x-2}{x^2-2}$ continuous?

2. Find the set where f is continuous, where

$$f(x) = \begin{cases} x+1 & x < -1 \\ x^2 - x & -1 \leq x < 1 \\ 2 & x = 1 \\ \frac{-x^2+3x-2}{x-2} & x > 1. \end{cases}$$

3. Find the set where h is continuous, where

$$h(x) = \begin{cases} \sin\left(\frac{1}{x}\right) & x \neq 0 \\ 0 & x = 0. \end{cases}$$

4. What is the average rate of change of the function $f(x) = 1 - x^2$ on the interval $[-1, 5]$?

5. Find the derivative of $f(x) = x - 3x^2$ at the point $x = 4$.

6. Write the equation of a line tangent to the curve $y = x^3 - x$ at the point $(1, 0)$.

7. What is the derivative of $f(x) = |x|$? (Hint: It's not defined everywhere.)

8. According to your textbook, the cost in dollars to produce x tacos is $C(x) = -\frac{3}{800}x^2 + \frac{3}{2}x + 1000$, for $0 \leq x \leq 180$. Find a formula for marginal cost at a level of x tacos. What is the marginal cost at a level of 100 tacos?

9. Find the derivative of $f(x) = \sqrt[3]{x}$. (Hint: For square roots, you multiply by the conjugate and take advantage of the fact that $(a-b)(a+b) = a^2 - b^2$. Try to do something analogous using $(a-b)(a^2+ab+b^2) = a^3 - b^3$.)

10. (a) Expand out $(x+h)^2$, $(x+h)^3$, and $(x+h)^4$.

    (b) You may already notice some patterns, but the only pattern we need for now is

    $$(x+h)^n = x^n + nx^{n-1}h + (\text{terms divisible by } h^2)$$

    Argue for why this equation must hold for all n. (Hint: There are many ways to go here, but one way is to count how many ways you can pick a term from each factor in $(x+h)(x+h)\cdots(x+h)$, so that the product comes out to $x^{n-1}h$.)

    (c) Use this to calculate the derivative of $f(x) = x^n$. (You may already know this rule from a previous class, but I want you to show why this rule works.)