# Week 3 Worksheet

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1. Convert 40° to radians. Convert $\frac{7\pi}{8}$ to degrees.

2. Complete the following table, filling in the sign for each function (if you get stuck, remember you can write everything in terms of sine and cosine!)

| Range | $\sin\theta$ | $\cos\theta$ | $\tan\theta$ | $\sec\theta$ | $\csc\theta$ | $\cot\theta$ |
|---|---|---|---|---|---|---|
| $0 < \theta < \pi/2$ | $+$ | | | | | |
| $\pi/2 < \theta < \pi$ | $+$ | | | | | |
| $\pi < \theta < 3\pi/2$ | $-$ | | | | | |
| $3\pi/2 < \theta < 2\pi$ | $-$ | | | | | |

3. Find the period and amplitude of the functions $f(x) = 10\cos(4x)$ and $g(x) = -2\sin\left(\frac{\pi}{4}x + 10\right)$.

4. For what values of $\theta$ does $\sin\theta = -1/2$ and $\cos\theta = \sqrt{3}/2$? (I'm asking for all values here, not just in the range $[0, 2\pi)$.)

5. Compute $\lim_{x\to 0^-} \frac{|x|}{x}$, $\lim_{x\to 0^+} \frac{|x|}{x}$ and $\lim_{x\to 0} \frac{|x|}{x}$.

6. Let $g$ be the function

$$g(x) = \begin{cases} 5 & x < 0 \\ x^2 - 2 & 0 \le x \le 3 \\ 7 & x > 3 \end{cases}$$

   Find $\lim_{x\to 0} g(x)$ and $\lim_{x\to 3} g(x)$.

7. Compute $\lim_{x\to\infty} \frac{9001x^2 + 4x}{x^3 - 1}$.

8. Compute $\lim_{x \to -\infty} g(x)$ and $\lim_{x \to \infty} g(x)$, where $g$ is the following function:

$$g(x) = \begin{cases} \frac{2x+1}{x-1} & x < 1 \\ x+3 & 1 \leq x < 2 \\ \frac{1}{x^2+1} & x \geq 2 \end{cases}$$

9. Compute $\lim_{x \to \infty} \dfrac{\sin x}{x}$

10. Does there exist a number $k$ such that $\lim_{x \to 2} \dfrac{3x^2 + kx - 2}{x^2 - 3x + 2}$ exists? Find such a $k$ or explain why it can't exist. Then do the same for $\lim_{x \to 2} \dfrac{x^2 + kx}{x^2 - 4x + 4}$.

11. In class, we learned $\lim_{x \to 0} \dfrac{\sin x}{x} = 1$. This challenge exercise will outline how to show this using elementary geometry and algebra.

    (a) Use geometry to argue that for small positive values of $x$,

    $$\sin x \leq x \leq \tan x.$$

    (How can we compare an angle to a length? Remember that radians measure arc length around the unit circle!)

    (b) Manipulate these inequalities using algebra to get

    $$\cos x \leq \frac{\sin x}{x} \leq 1$$

    (c) Compute the limit of $\cos x$ as $x \to 0$. Give an intuitive argument why this means $\frac{\sin x}{x}$ must converge to 1 as $x \to 0^+$. (We only get from the right, since in the first step we assumed $x$ was positive to get the geometry right).

    (d) Adapt this argument for negative values of $x$ to get the two-sided limit.