Week 2 Worksheet

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1. Solve for x in $5^{x/2} = 100$.

2. What is the largest possible domain for the function $f(x) = \frac{1}{\sqrt{1-x^2}}$? What about for $g(x) = \sqrt{1-x^2}$?

3. Find the domain for the function
$$f(x) = \sqrt{\frac{x^3 + 2x^2 + x}{x - 5}}.$$

4. Label each of the following functions as *even*, *odd*, *both*, or *neither*:
   (a) $f(x) = x^5 + x^3 + x$
   (b) $g(x) = 5$
   (c) $h(x) = x^3 + x^2$

5. Give an example of a function which is both even and odd. Do any other examples exist? Justify your answer.

6. You remember learning in a physics class that when you throw a ball in the air, its height (in meters) as a function of time (in seconds) $h(t)$ can be modeled by a quadratic. Unfortunately, you completely forget what this quadratic is. As an experiment, you throw a ball in the air. It travels to a maximum height of 4.9 meters and lands back in your hand after 2 seconds. Use this information to write an expression for $h(t)$.

7. European paper comes in "A" sizes: A0, A1, A2, etc. A0 paper is huge and is used for posters, while A4 paper is close to "normal" US Letter paper. When you cut a peice of A0 paper in half, you get two A1 sheets. When you cut an A1 sheet, you get two A2 sheets, and so on.

At the same time, the ratio of width to height w/h remains the same each time you cut it in half. What must this ratio be? (**Hint:** The tricky part here is setting things up. Try drawing a picture. At the end of the day you will be solving a quadratic.)

8. In music, the note A4 has a frequency of 440 Hz. The smallest gap between notes is called a *half-step*, and 12 half-steps make an *octave*. Every time you move up an octave, the frequency of a note doubles. Write an function $f(s)$, giving the frequency (in Hz) of a note as a function of the number of half-steps s above A4. The note C4 (often called *middle C*) is 9 half-steps below A4. What is its frequency? (**Hint:** First think about frequency as a function of octaves above A4, and then octaves as a function of half-steps, and then put them together.)

9. In many disciplines, it can be useful to plot data or a function on a grid where the axes don't represent the values x and y directly, but rather $\log_{10} x$ and $\log_{10} y$. In this exercise, we will explore what this transformation does to a few familiar functions. Let u and v be the points on this grid, so $u = \log_{10} x$ and $v = \log_{10} y$. If we want to go the other way, we have $x = 10^u$ and $y = 10^v$.

   To see what $y = 5x$ looks like on a log-log plot, re-write it in terms of u and v, and write v as a function of u.

   What does $y = 1/x$ look like on this plot?

   A function that looks linear on a log-log plot is of the form $v = mu + b$, or equivalently, $\log_{10} y = m \log_{10} x + b$. To see what function this represents, write this as a function of y and simplify.

   If a function looks like $v = u^2$ on a log-log plot, what was the original function y in terms of x?