Week 12 Worksheet

James Hagborg

26 November 2019

1. (a) Use a Riemann sum with $n = 4$, choosing smaple points at midpoints, to find the area between the function $f(x) = \sqrt{4 - x^2}$ and the x-axis.

   (b) Using geometry, what do you expect $\int_{-2}^{2} f(x)dx$ to be? (I'm asking you to draw the picture and think about what this quantity means. You do not need limits, derivatives, antiderivatives, etc.)

2. Using the fundamental theorem of calculus, compute the following definite integrals:

   (a) $\displaystyle\int_0^{10} (x^2 + x - 10)dx$

   (b) $\displaystyle\int_{-1}^{1} \frac{10}{2 - x}dx$

   (c) $\displaystyle\int_1^{e} \frac{1}{x(\ln x)}dx$

3. True or false: if f is an odd function and $a > 0$, then $\int_{-a}^{a} f(x)dx = 0$.

4. Compute the area between the curves $y = \sqrt{x}$ and $y = x\sqrt{x}$.

5. A company introduces a new machine which produces savings at a rate of

$$S'(t) = 150 - t^2$$

   dollars per year. The machine will cost

$$C'(t) = t^2 + \frac{11}{4}t$$

dollars per year. How long is it worth it to operate the machine? Over this period of time, how much will it save?

6. Write down a function f such that $f'(x) = e^{-x^2}$ and $f(2) = 7$. (You are allowed to keep the "$\int$" symbol in your answer.)

7. Write down a function f which is differentiable at $x = 0$, but whose derivative is not differentiable at $x = 0$. (You could have done this before, but the FTC makes it much easier.)