Week 10 Worksheet

James Hagborg

5 November 2019

1. Find the absolute extrema if they exist, as well as all values of x where they occur.

    (a) $f(x) = x^3 - 2x^2 - 4x + 8$ on $[-1, 0]$.

    (b) $h(x) = xe^{-kx}$ on $[0, \infty)$, for a constant $k > 0$.

    (c) $k(x) = \sin(x)$ on $(-\infty, \infty)$.

    (d) $\ell(x) = \begin{cases} x^2 & x \geq 0 \\ 0 & x < 0 \end{cases}$ on $(-\infty, \infty)$.

    (e) $m(x) = x\sin(x)$ on $[0, \infty)$.

2. A hunter is at a point along the river bank. He wants to get to his cabin, located 3 miles north and 8 miles west. He can travel west along the river at the speed of 5 mph, but only travel 2 mph through the wilderness between the river and his cabin. How far upriver should he travel in order to reach the cabin in minimum time?

3. Determine whether the demand is elastic or not at the indicated values: $q = 400 - 0.2p^2$, for $p = \$20$ and $p = \$40$.

4. Suppose we want to cut a rectangular beam from a cylindrical log of radius 10 inches.

    (a) Show that the beam of a maximal cross sectional area is a square.

    (b) Suppose the strength of a beam is proportional to the product of its width and the square of its depth (width and depth are two sides of the cross section rectangle). Find the dimension of the strongest beam that can be cut from the log.