

Probabilistic Operator Algebra Seminar

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Title: *Sums of commutators in free probability*

In their study of the free commutator [6] A.Nica and R.Speicher observed that odd cumulants cancel, i.e., they do not appear in the universal formula expressing the moments of the commutator in terms of the moments of the individual operators.

More recently, O.Arizmendi, T.Hasebe, and N. Sakuma [1] observed that the free commutator preserves free infinite divisibility.

In ongoing joint work with W.Ejsmont on quadratic forms [2, 4, 5, 3] we could show that this is no coincidence and that in fact for a quadratic form Q in noncommuting variables the following properties are equivalent:

1. preservation of free infinite divisibility
2. free cancellation of odd cumulants
3. strong cancellation of odd cumulants
4. Q can be written as a linear combination of commutators

On the other hand, the distributions of quadratic forms in free i.d. symmetrically distributed random variables are computable. We illustrate this by showing that the tangent function arises as the R-transform of a free central limit law for sums of commutators.

Moreover, the free cumulants of finite sums of commutators of semicircular elements have a simple expression in terms of tangent and arctangent numbers. As a side result we found a simple evaluation of sums of powers of cotangents.

References

- [1] Octavio Arizmendi, Takahiro Hasebe, and Noriyoshi Sakuma, *On the law of free subordinators*, ALEA Lat. Am. J. Probab. Math. Stat. **10** (2013), no. 1, 271–291.
- [2] Wiktor Ejsmont and Franz Lehner, *Sample variance in free probability*, J. Funct. Anal. **273** (2017), no. 7, 2488–2520.
- [3] ———, *The free tangent law*, 2020, preprint, arXiv:2004.02679.
- [4] ———, *Sums of commutators in free probability*, 2020, submitted, arXiv:2002.06051.
- [5] ———, *The trace method for cotangent sums*, 2020, preprint, arXiv:2002.06052.
- [6] Alexandru Nica and Roland Speicher, *Commutators of free random variables*, Duke Math. J. **92** (1998), no. 3, 553–592.