

Probabilistic Operator Algebra Seminar

Organizer: Dan-Virgil Voiculescu

October 4 **Alexandru Nica and Daniel Perales**, University of Waterloo

Title: *Multiplicative and semi-multiplicative functions on non-crossing partitions, and relations to cumulants*

The idea of studying multiplicative functions in an incidence algebra goes back to the work of Rota and his collaborators in the 1960s. In the 1990s it was discovered by Speicher that, in the framework of lattices of non-crossing partitions, such multiplicative functions are a very useful tool for the combinatorial side of free probability. In this talk we present a joint work with A. Celestino, K.Ebrahimi-Fard and L. Witzman (arXiv:2106.16072), which starts from the following observation: when omitting one of the assumptions included in the definition of a multiplicative function, one gets a framework of “semi-multiplicative functions on non-crossing partitions”, which can be used to streamline calculations involving moment-cumulant and inter-cumulant formulas. This covers some brands of cumulants commonly used in non-commutative probability: free, Boolean, and monotone.

In the first part of the talk we give the description of the above framework, featuring a convolution group of semi-multiplicative functions on non-crossing partitions. For illustration, we discuss in detail a one-parameter interpolation between free and Boolean cumulants, the t -Boolean cumulants, arising from work of Bozejko and Wysoczanski.

In the second part of the talk we look at a dual picture for our framework. This is provided by a Hopf algebra, \mathcal{T} , such that the group of semi-multiplicative functions from part 1 is the character group of \mathcal{T} . We observe the analogy to the known fact that the corresponding group of multiplicative functions can be identified as the character group of the Hopf algebra Sym of symmetric functions. We show that, in fact, the inclusion of multiplicative functions into semi-multiplicative ones is the dual of a natural surjective homomorphism from \mathcal{T} onto Sym, provided by the Kreweeras complementation map on non-crossing partitions. The antipode of \mathcal{T} is a promising inversion tool for the general study of formulas which relate different brands of cumulants.