Probabilistic Operator Algebra Seminar

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March 14 **Zakhar Kabluchko**, University of Muenster

Title: *Repeated differentiation and free unitary Poisson processes*

We investigate the hydrodynamic behavior of zeros of trigonometric polynomials under repeated differentiation. We show that if the zeroes of a real-rooted, degree d trigonometric polynomial are distributed according to some probability measure ν in the large d limit, then the zeroes of its $[2td]$-th derivative, where $t > 0$ is fixed, are distributed according to the free multiplicative convolution of ν and the free unitary Poisson distribution with parameter t. In the simplest special case, this result states that the zeroes of the $[2td]$-th derivative of the trigonometric polynomial $(\sin\frac{\theta}{2})^{2d}$ are distributed according to the free unitary Poisson distribution with parameter t, in the large d limit. The latter distribution can be defined in terms of the function $\zeta = \zeta_t(\theta)$ which solves the implicit equation $\zeta - t\tan\zeta = \theta$ and satisfies

$$\zeta_t(\theta) = \theta + t\tan(\theta + t\tan(\theta + \cdots)).$$