

# Probabilistic Operator Algebra Seminar

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Title: *Connections between free probability and model theory of tracial  $W^*$ -algebras.*

We explain several intriguing connections between free probability and model theory of tracial von Neumann algebras studied by Farah, Hart, and Sherman, as well as a lot of open questions. First, we remark that Ben Yaacov showed that diffuse classical probability spaces admit quantifier elimination, which provides a model-theoretic heuristic why classical probability theory is much easier than non-commutative probability theory. Another consequence of this is that in the diffuse classical probability setting, the model theoretic type of a tuple of random variables  $(X_1, \dots, X_d)$  is uniquely determined by its probability distribution (which corresponds to its quantifier-free type). On the other hand, in tracial  $W^*$ -algebras, the non-commutative law does not determine the model-theoretic type. We propose that the non-commutative law (or quantifier-free type) and the full type could both be considered analogs of classical probability distributions. Hence, we also describe a version of free entropy theory for full types, and as an application show that a matrix ultraproduct is not strongly 1-bounded. Moreover, we sketch potential ways that the model-theoretic framework could help us address problems related to optimal couplings and differential equations in free probability.