Probabilistic Operator Algebra Seminar

Organizer: Dan-Virgil Voiculescu

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Title: *Free entropy theory for types in tracial W^*-algebras.*

We describe an adaptation of the theory of three entropy (and 1-bounded entropy in particular) to model-theoretic types of tuples in a tracial W^*-probability space. Roughly speaking, the theory of micro states free entropy quantifies the amount of matrix approximations for non-commutative random variables $X = (X_1, X_2, ...)$ in a tracial W^*-probability space. "Matrix approximations" traditionally means matrix tuples with approximately non-commutative moments as X. But rather than only using polynomials as test functions, we can study the values of arbitrary formulas obtained by combining traces of non-commutative polynomials with supremum and infimum operations over auxiliary variables in an operator-norm ball. The values of such formulas are encoded by the type of X, a notion from continuous model theory. We can define "microstate spaces" of matrix approximations for types, and thus obtain analogs of Voiculescu's free entropy and free entropy dimension (by looking at the asymptotic of the Lebesgue measure of these spaces) and of Jung and Hayes' 1-bounded entropy (by looking at the asymptotic of the covering numbers of these spaces). We will describe how this free entropy theory for types relates to the free entropy for X and free entropy for X in the presence of Y, showing that these variants arise naturally from quantifier-free and existential formulas in model theory. We also present variational principles relating the entropy for full types with the entropy for existential types, and use this to deduce several results about embeddings of tracial W^*-algebras into a matrix ultra product. Finally we present some open questions for further research.