Probabilistic Operator Algebra Seminar

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January 31 **Malte Gerhold**, University of Greifswald

Title: *Towards a classification of multifaced independences*

Bi-freeness was the first independence for pairs of faces, and quickly after its invention by Voiculescu the question arose whether there are other natural independences in the two-faced setting. Two directions were pursued: Gu and Skoufranis defined c-bi-free independence, a two-faced two-state independence from which they derive bi-boolean and (with Hasebe) bi-monotone independence, paralleling Bozejko and Speicher's c-free independence in the single-faced case. Another approach is to search for natural products of representations, which led Liu to free-boolean independence and myself to another notion of bi-monotone independence (with a monotone and an anti monotone face). The merit of working with representations is that it is easy to establish that the corresponding product of states is again a state, a property which fails for c-bi-free independence and its derivates. In this talk we will take a closer look at the representation theoretic approach to independence for pairs (or larger families) of faces. We present axioms for universal products of representations which assure that they induce well behaved independences and also lead to surprising new examples. By restricting to the tensor product or free product as representation space for the product representation we define subclasses of multi-faced independences which we can completely classify. If time permits, we will also discuss a third approach based on moment-cumulant formulae that was investigated by Philipp Varso and leads to a number of two-faced independences with interesting combinatorics for which proving or disproving positivity is an open problem at the moment. The talk will be based on work with Takahiro Hasebe and Michael Ulrich, in particular our recent preprint arXiv:2111.07649.