Probabilistic Operator Algebra Seminar

Organizer: Dan-Virgil Voiculescu

June 30 **David Jekel and Kyeongsik Nam**, UCSD and UCLA

Title: *Duality for optimal couplings in free probability*

We study the free probabilistic analog of optimal couplings for the quadratic cost, where classical probability spaces are replaced by tracial von Neumann algebras and probability measures on $\mathbb{R}^m$ are replaced by non-commutative tracial laws of m-tuples. We prove an analog of the Monge-Kantorowich duality which characterizes optimal couplings of non-commutative tracial laws with respect to Biane and Voiculescu's noncommutative L^2-Wasserstein distance using a new type of convex functions. As a consequence, we show that if (X, Y) is a pair of optimally coupled m-tuples of non-commutative random variables in a tracial W^*-algebra $\mathcal{A}$, then $W^*((1-t)X+tY) = W^*(X, Y)$ for all $t \in (0, 1)$. Finally we illustrate the subtleties of non-commutative optimal couplings through connections with results in quantum information theory and operator algebras. For instance, two non-commutative laws that can be realized in finite-dimensional algebras may still require an infinite-dimensional algebra to optimally couple. Moreover, the space of non-commutative laws of m-tuples is not separable with respect to the Wasserstein distance for $m > 1$.