

Probabilistic Operator Algebra Seminar

Organizer: Dan-Virgil Voiculescu

April 26 **Colin R. Defant**, Princeton University

Title: *Troupes, Cumulants, and Stack-Sorting*

Cumulants are fundamental combinatorial tools in noncommutative probability theory. We will discuss a formula that converts from a sequence of free cumulants to the corresponding sequence of classical cumulants via a sum over combinatorial objects called valid hook configurations. These objects also appear in a formula that counts preimages of permutations under a map called the postorder traversal. By combining these formulas, we can use the postorder traversal to explain a surprising phenomenon concerning sets of colored binary plane trees that we call troupes. Namely, if a sequence of free cumulants counts the trees in the troupe, then the corresponding sequence of classical cumulants counts decreasing labeled versions of trees in the troupe. In the opposite direction, we will see that tools from combinatorial noncommutative probability theory provide a powerful method for proving deep facts about the postorder traversal and a closely related function called the stack-sorting map.