

Probabilistic Operator Algebra Seminar

Organizer: Dan-Virgil Voiculescu

February 21 **Gaëtan Borot and Elba Garcia-Failde**, Humboldt
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Title: *Analytic theory of higher order freeness*

We introduce a notion of surfaced free cumulants from the combinatorics of surfaced permutations (which is a generalization of partitioned permutations). We express the universal relation between surfaced moments and surfaced free cumulants in terms of an operation related to double monotone Hurwitz numbers. Using the semi-infinite wedge formalism and techniques recently developed by Bychkov, Dunin-Barkowski, Kazarian and Shadrin, we can deduce functional relations between the corresponding generating series. In the genus 0 sector: at order 1 we retrieve the usual notion of free cumulants and the R -transform machinery of Voiculescu; at order 2 we retrieve the second order R -transform of Collins, Mingo, Sniady and Speicher; at higher order we retrieve their notion of higher order free cumulants for which obtaining the functional relations was an open problem. The theory is however simplified by including all genera, and leads to a notion of surfaced freeness. While the genus 0 truncation captured only the large N limit, the surfaced moments/free cumulants capture the all-order asymptotic expansion in $1/N$ of random ensembles of matrices of size N in presence of some unitary invariance. This is joint work with Séverin Charbonnier, Felix Leid and Sergey Shadrin: <https://arxiv.org/abs/2112.12184>