Math 55 Section Worksheet

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1 Warm-Up

(a) How many ways can I give 3 distinct candies to 5 children? (some children may receive none and some may receive more than 1)

(b) How many ways can I give 3 distinct candies to 5 children if no child is allowed more than 1 candy?

(c) How many ways can I give 3 indistinguishable candies to 5 children if no child is allowed more than 1 candy?

(d) How many ways can I give 3 indistinguishable candies to 5 children, if children are allowed more than 1 piece of candy?

2 Let's Count!

1. How many ways are there to choose a dozen donuts from the 24 varieties at Kingpin Donuts?
2. How many different strings can be made from the letters in MISSISSIPPI, using all the letters?
3. How many solutions are there to the equality $x_1 + \ldots + x_k = n$, where $x_i, n \in \mathbb{N}$?
4. How many solutions are there to the inequality $x_1 + \ldots + x_k \leq n$?
5. A *lattice path to* (a, b) is a walk starting at the origin and ending at (a, b) where at each step you are allowed to move one unit north or one unit east (see 6.4 #33 on your homework).
   (a) How many lattice paths are there to $(2n, 2n)$?
   (b) How many lattice paths are there to $(2n, 2n)$ that go through (n, n)?
6. Count the number of 6 card hands dealt from a standard deck of 52 cards that have at least one card in every suit.
7. How many ways can n books be placed on k distinguishable shelves
   (a) if the books are indistinguishable copies of the same title?
   (b) if no two books are the same, and the positions of the books on the shelves matter?

3 Bonus

The following is called the *twelvefold way* in combinatorics. Let X, N be finite sets of size x, n, respectively. Below, "in/distinguishable" means that the elements of the set either can or cannot be distinguished from each other. They are still distinct elements, there just might not be a difference between them. (think k different-brand candies versus k candies all of the same brand). Try to fill out the marked entries of the table:

	# of $f : N \to X$	# of injective $f : N \to X$	# of surjective $f : N \to X$
N distinguishable X distinguishable	(a)	(b)	
N indistinguishable X distinguishable	(c)	(d)	(e)
N distinguishable X indistinguishable		(f)	
N indistinguishable X indistinguishable		(g)	

Bonus: Try to fill out the remainder of the table.