Math 55 Section Worksheet

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1 Warm-Up

Try to recall the following concepts *without* looking at your notes:
r-permutation *r-combination* *binomial coefficient*
binomial theorem $\binom{n}{k}$

2 Together

1. Find the number of 5-permutations of a set with nine elements.
2. Let $n \in \mathbb{N}$. Prove by a combinatorial argument that

$$\sum_{k=0}^{n} 2^k \binom{n}{k} = 3^n$$

3 You Try

3. A professor writes 40 discrete mathematics true/false questions. Of the statements in these questions, 17 are true. If the questions can be positioned in any order, how many different answer keys are possible?
4. The English alphabet contains 21 consonants and 5 vowels. How many strings of 6 English letters contain
   (a) exactly one vowel?
   (b) exactly two vowels?
   (c) at least one vowel?
   (d) at least two vowels?
5. What is the coefficient of x^4y^7 in the expansion of $(2x - y)^{11}$?
6. Let $n \in \mathbb{N}$. Prove by a combinatorial argument that

$$\binom{2n}{2} = 2\binom{n}{2} + n^2$$

7. Let $n \in \mathbb{Z}^+$. Prove by a combinatorial argument that

$$n \cdot 2^{n-1} = \sum_{k=1}^{n} \binom{n}{k} \cdot k$$