

Math 55 Section Worksheet

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1 Warm-up

Try to recall the following concepts *without* looking at your notes:

Sum Rule *Product Rule* *Inclusion-Exclusion* *Pigeonhole Principle*

2 Problems

1. What is wrong with this “proof” by strong induction?
“*Theorem*” For every nonnegative integer n , $5n = 0$.
Base Case: $5 \cdot 0 = 0$.
Inductive Step: Suppose that $5j = 0$ for all nonnegative integers j with $0 \leq j \leq k$. Write $k + 1 = i + j$, where i and j are natural numbers less than $k + 1$. By the inductive hypothesis, $5(k + 1) = 5(i + j) = 5i + 5j = 0 + 0 = 0$.
2. Suppose you begin with a pile of n stones and split this pile into n piles of one stone each by successively splitting a pile of stones into two smaller piles. Each time you split a pile you multiply the number of stones in each of the two smaller piles you form, so that if these piles have r and s stones in them, respectively, you compute rs . Show that no matter how you split the piles, the sum of the products computed at each step equals $n(n - 1)/2$.
3. Prove that $2^n > (n + 1)^2$ for all $n \in \mathbb{N}$ with $n \geq 6$.
4. Give a recursive definition of $P_m(n)$, the product of the integer m and the nonnegative integer n .
5. How many different three-letter initials can people have?
6. How many different three-letter initials with none of the letters repeated can people have?
7. How many positive integers not exceeding 100 are divisible either by 4 or by 6?
8. Let $n \in \mathbb{N}$ and suppose we have a set $S \subseteq [2n]$ of size $|S| = n + 1$. Prove that there must be two elements $x, y \in S$ that are relatively prime.