# Math 55 Section Worksheet

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## 1 Warm-up

1. What is the difference between Induction and Strong Induction?
2. What is the Well-Ordering Property?

## 2 Problems

1. Prove that for all $n \in \mathbb{N}$ with $n \geq 1$,

$$\frac{1}{1 \cdot 2} + \frac{1}{2 \cdot 3} + \frac{1}{3 \cdot 4} + \cdots + \frac{1}{n(n+1)} = \frac{n}{n+1}$$

2. Let $f : \mathbb{R} \to \mathbb{R}$ be such that $f(xy) = xf(y) + yf(x)$. Prove that for all $u \in \mathbb{R} - \{0\}$ and all $n \in \mathbb{N} - \{0, 1\}$, $f(u^n) = nu^{n-1}f(u)$.
3. Consider the Fibonacci sequence $f_n$ defined by setting $f_0 = 0, f_1 = 1$ and for $n \geq 2$, $f_n = f_{n-1} + f_{n-2}$. Prove the following:
   (a) $\sum_{i=0}^{n} f_i = f_{n+2} - 1$
   (b) $\sum_{i=0}^{n} f_i^2 = f_n \cdot f_{n+1}$
   (c) $f_{n-1} \cdot f_{n+1} - f_n^2 = (-1)^n$
4. Prove that $n^3 + 2n$ is divisible by 3 for all integers $n$.
5. Which amounts of money can be formed using just two-dollar bills and five-dollar bills?
6. Prove that a convex $n$-gon has $n(n-3)/2$ diagonals.