

Math 55 Section Worksheet

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1 Questions

Think of questions to ask!

2 Midterm Review

1. Let A, B be sets and P, Q, R propositions. Negate the following proposition:

$$\exists x \in A \forall y \in B \forall z \in A, P(x, y, z) \rightarrow (Q(z) \wedge R(x))$$

2. Let $f : \mathbb{Z} \rightarrow \mathbb{Z}$ be defined by $f(x) = 2x - 3$. Let $g : \mathbb{Z} \rightarrow \mathbb{N}$ be defined by $g(x) = |x| + 4$. What is the domain of $g \circ f$? What is the codomain? Is this function injective? Surjective?

3. Let \mathbb{R} be the universe and $P(x, y)$ be the statement $x + y \in \mathbb{Q}$. Determine the truth values of the following statements:

(a) $\forall x \in \overline{\mathbb{Q}} \exists y \in \overline{\mathbb{Q}} P(x, y)$

(b) $\exists x \in \overline{\mathbb{Q}} \forall y \in \overline{\mathbb{Q}} P(x, y)$

4. Evaluate $(3^{99} + 6^4) \bmod 8$.
5. Is $\{2, \emptyset\} \subseteq \mathcal{P}(\mathbb{Z} \cup \{\emptyset\})$?
6. Let $f : \mathbb{N} \times \mathbb{N} \rightarrow \mathbb{N}$ be defined by $f(x, y) = 2^{x-1}(2y - 1)$. Prove that f is injective.
7. Let $a, b, c \in \mathbb{Z}$ such that $a^2 + b^2 = c^2$. Prove that at least one of a, b is even. (Hint: Look at $c^2 \bmod 4$).