Math 55 Section Worksheet GSI: Jeremy Meza Office Hours: Wed 10-12pm, Evans 775 February 7, 2018

## 1 Warm-up

- 1. What is the definition of gcd(a, b)?
- 2. What is the definition of *relatively prime*?
- 3. State Bezout's Theorem.

## 2 Problems

- 1. What is the base 3 expansion of 172?
- 2. Evaluate the following:
  - (a)  $(19^2 \mod 41) \mod 9$
  - (b)  $(32^3 \mod 13)^2 \mod 11$
  - (c)  $(7^3 \mod 23)^2 \mod 31$
  - (d)  $(21^2 \mod 15)^3 \mod 22$
- 3. Calculate gcd(224, 126). Write the gcd as a linear combination of 224 and 126.
- 4. Let  $a, b, c, m \in \mathbb{Z}$ . Prove that if  $a \equiv b \pmod{m}$ , then  $ac \equiv bc \pmod{m}$ . Is it true that if  $ac \equiv bc \pmod{m}$ , then  $a \equiv b \pmod{m}$ ?

## 3 Extra

- 5. Let  $a, b, c \in \mathbb{Z}$  such that  $a^2 + b^2 = c^2$ . Prove that at least one of a, b is even. (Hint: Look at  $c^2 \mod 4$ ).
- 6. Let A be a palindromic positive integer with an even number of digits. For example, A could be 403,304. Prove that A is divisible by 11. (Hint: write A out in base 10 expansion).