Math 55 Section Worksheet

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1 Warm-up

1. What is the definition of countable?
2. What is the definition of $a \mid b$?
3. What is the definition of $a \equiv b \pmod{m}$?
4. Evaluate $-23 \pmod{4}$.
5. True or False: If $a \mid (mn)$ then either $a \mid m$ or $a \mid n$.

2 Problems

1. Let S be the set of all infinite binary strings (that is, elements of S are infinitely long strings of 0s and 1s). Provide a bijection between S and $\mathcal{P}(N)$.
2. Let $[n]$ denote the set $\{1, \ldots, n\}$. Let A be the set of subsets of $[n]$ that have even size and B be the set of subsets of $[n]$ that have odd size. Establish a bijection between A and B.
3. Let $a, b, c \in \mathbb{Z}$. Prove that if $a \mid b$ and $b \mid c$, then $a \mid c$.
4. What is the base 5 expansion of 154?

3 Extra

5. Prove $\mathbb{N} \times \mathbb{N}$ is countable. Hint: draw a diagram.
6. Let $a, b, c \in \mathbb{Z}$ such that $a^2 + b^2 = c^2$. Prove that at least one of a, b is even. (Hint: Look at c mod 4 and mod 2).
7. Let A be a palindromic positive integer with an even number of digits. For example, A could be $403,304$. Prove that A is divisible by 11. (Hint: write A out in base 10 expansion).
8. Let A, B be countable sets.
   (a) Prove that if $A \cap B = \varnothing$, then $A \cup B$ is countable.
   (b) Prove in general that $A \cup B$ is countable.

9. What do you think are the cardinalities of the following sets?

   (a) The set of all infinite binary strings.
   (b) The set of all infinite binary strings with only *finitely* many 1s.
   (c) The set of functions from $\mathbb{N}$ to $\mathbb{N}$.
   (d) The set of functions from $\mathbb{N}$ to $\{1, 2\}$.
   (e) $\bigcup_{n \in \mathbb{N}} \{0, 1, \ldots, n\}$.
   (f) $\bigcup_{x \in \mathbb{R}} \{x\}$.
   (g) The rational numbers.
   (h) The irrational numbers.
   (i) $\mathbb{N} \times \mathbb{Z} \times \mathbb{Q}$.
   (j) $\mathcal{P}(\mathbb{N})$.
   (k) $\mathcal{P}(\mathbb{R})$.
   (l) $\mathcal{P}(\mathcal{P}(\mathbb{N}))$.
   (m) The number of different cardinalities.
   (n) The number of bijections from $[n]$ to itself.
   (o) $\mathbb{C}$
   (p) Any set that contains an uncountable set.
   (q) Any set contained in a countable set.
   (r) Any set that has a surjection onto an uncountable set.
   (s) Any set that has an injection into a countable set.
   (t) All the grains of sand in the world.
   (u) All the quarks in all the protons in all nuclei in all the atoms in all the particles in all of the universe.
   (v) A basis of $\mathbb{R}$ as a vector space over $\mathbb{Q}$.
   (w) The set of all polynomials with integer coefficients.
   (x) The set of algebraic numbers.
   (y) The set of transcendental numbers.
   (z) The set of computable numbers.