Math 55 Section Worksheet

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April 18, 2018

1 Warm-Up

Try to recall the following concepts *without* looking at your notes.

graph	directed graph	degree	"Handshaking" Theorem
complete graph	cycle graph	bipartite graph	adjacency matrix

2 Problems

1. How many vertices and edges are in the following graphs:
   (a) The complete graph K_n.
   (b) The cycle graph C_n.
   (c) The wheel graph W_n.
   (d) The n-cube Q_n.
2. For which values of n are the following graphs bipartite?
   (a) The complete graph K_n.
   (b) The cycle graph C_n.
   (c) The wheel graph W_n.
   (d) The n-cube Q_n.
3. Think of questions to ask!

3 Challenge

4. A graph is *planar* if it can be drawn without any edges intersecting (edges don't have to be straight lines). Which of the following graphs do you think are planar?

   (a) K_3? K_4? K_5? (b) $K_{2,2}$? $K_{3,3}$?

5. The *Ramsey number* $R(r, s)$ is the minimum number of vertices n such that no matter how we color the edges of the complete graph K_n with colors red and blue, there is always either a red K_r or a blue K_s. For example, we have shown previously that $R(3, 3) = 6$. (Why?)

   Easy: Show that $R(2, s) = s$.
   Hard: Show that $R(4, 3) \leq 10$.
   Harder: Show that $R(r, s) \leq R(r-1, s) + R(r, s-1)$ (Hint: consider the complete graph on $N = R(r-1, s) + R(r, s-1)$ vertices. Do a similar pigeonhole argument.)