Math 55 Section Worksheet

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Office Hours: Wed 10am-12pm, Evans 775
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1 Warm-Up

(a) What is a *binary relation*? What does it mean to be reflexive, symmetric, antisymmetric, and transitive?

(b) What is the $\mathcal{P}$*-closure* of a relation?

(c) What is an *equivalence relation*? What is an *equivalence class*?

2 Problems

1. Let R be an equivalence relation on a set A. If $a\mathrm{R}b$, prove that $[a] = [b]$.

2. Consider the relation on $\mathbb{R} \times \mathbb{R}$ given by

$$(a, b)\ \mathrm{R}\ (c, d) \iff a = c$$

Is this an equivalence relation? What are the equivalence classes? Try to visualize it on the plane. Do the equivalence classes partition $\mathbb{R} \times \mathbb{R}$?

3. Let R be a relation on A. Prove by induction that there is a path of length n, where $n \in \mathbb{Z}^+$, from a to b iff $(a, b) \in \mathrm{R}^n$. (Recall R^n is the composite relation of R with itself, n times.)

4. Define a relation R on $\mathbb{N} \times \mathbb{N}$ by

$$(a, b)\ \mathrm{R}\ (c, d) \iff a - b = c - d$$

Prove that R is an equivalence relation. What do you think are the equivalence classes?

5. Define a relation R on $\mathbb{Z} \times (\mathbb{Z} \setminus \{0\})$ by the following:

$$(a, b)\ \mathrm{R}\ (c, d) \iff ad = bc$$

Prove that R is an equivalence relation. What do you think are the equivalence classes?

6. Let $S = \{(x_1, \ldots, x_k) \subseteq [n]^k \mid \forall i \neq j,\ x_i \neq x_j\}$. Define a relation R on S by $(x_1, \ldots, x_k)$ R $(y_1, \ldots, y_k)$ iff $(y_1, \ldots, y_k)$ is some permutation of $(x_1, \ldots, x_k)$.

   (a) Prove that R is an equivalence relation.
   (b) For $(x_1, \ldots, x_k) \in S$, what is the size of $[(x_1, \ldots, x_k)]$?
   (c) How many equivalence classes are there?
   (d) What is the size of S?