

Math 55 Section Worksheet

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April 4, 2018

1 Warm-Up

Try to recall the following concepts *without* looking at your notes.

binary relation reflexive relation transitive relation directed graph
symmetric relation antisymmetric relation composite relation

2 Problems

1. Define the relation R on \mathbb{Z} by $(x, y) \in R \iff 3 \mid x - y$.

- (a) Prove that R is reflexive.
- (b) Prove that R is symmetric.
- (c) Prove that R is transitive.

2. Give an example of a relation on a set that is

- (a) both symmetric and antisymmetric.
- (b) neither symmetric nor antisymmetric.

3. Let R, S be the relations represented by the matrices

$$M_R = \begin{pmatrix} 0 & 1 \\ 1 & 1 \end{pmatrix} \quad M_S = \begin{pmatrix} 1 & 1 \\ 1 & 0 \end{pmatrix}$$

Find the matrix that represents the composite relation $R \circ S$. Draw the directed graphs for R, S , and $R \circ S$.

4. Let $n \in \mathbb{N}$. Consider the function $\varphi: [n] \times \mathbb{R}^n \rightarrow \mathbb{R}^n$ defined by

$$\varphi(a, (x_1, \dots, x_n)) = (x_{n-a+1}, \dots, x_n, x_1, x_2, \dots, x_{n-a})$$

- (a) In words, describe what φ does to the ordered list (x_1, \dots, x_n) .
- (b) Define a relation R on \mathbb{R}^n by

$$(x, y) \in R \iff \exists a \in [n], \varphi(a, x) = y$$

Prove that R is reflexive, symmetric, and transitive.

3 Challenge

5. How many relations are there on a set with n elements that are
- (a) symmetric?
 - (b) reflexive?
 - (c) reflexive and symmetric?
 - (d) antisymmetric?
 - (e) asymmetric?