Math 55 Section Worksheet

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1 Warm-Up

Try to recall the following concepts *without* looking at your notes.

random variable	expected value	geometric distribution
independent random variables	variance	standard deviation

2 Together

1. What is the expected number of times a 6 appears when a fair die is rolled 10 times?

3 You Try!

2. Suppose that you flip a fair coin n times.
   (a) What is the expected number of heads?
   (b) What is the variance of the number of heads?

3. You've just gotten your wisdom teeth pulled and have recently exited the dentist's office still delirious from anesthesia. You make it back to your apartment, pull out your keychain of n keys, but can't quite manage to figure out which key is the right one. So, you try each key one by one. What is the expected number of selections you will make until you are soundly asleep in your apartment if
   (a) you choose the keys without replacement?
   (b) you choose the keys with replacement?

4. Let X_n be the random variable that equals the number of tails minus the number of heads when n fair coins are flipped.
   (a) What is the expected value of X_n?
   (b) What is the variance of X_n?

5. The math department has just hired several new GSIs for Math 55 so that there are now n students and n GSIs for the course. Moreover, Professor Williams has graciously decided to make the next exam a joint exam; she randomly pairs everyone together and they will take the exam together (these pairings can be student-student, student-GSI or GSI-GSI). What is the expected number of student-GSI pairings?

4 Challenge

6. Consider repeatedly flipping a coin in which the probability of heads is p. The flips are independent from each other. Let T be the random variable that equals the number of coin flips until you get a heads. Let S be the random variable that equals the number of coin flips until you get your second heads. Let R be the random variable $S - T$.

   (a) Compute the expected values of S and R.

   (b) Compute the variance of S.

   (c) Show that the pair of events $T = k$ and $R = j$ are independent for every choice of k, j.

7. Assume we have n distinct points $V = \{v_1, v_2, \ldots, v_n\}$ on the perimeter of a circle. For each $1 \leq i < j \leq n$, we flip a biased coin with probability p of heads and we connect v_i with v_j with a straight line segment (an edge) if and only if we get a head. The resulting random "network" is called a *random graph* with vertex set V . Show that

   (a) the expected number of edges in the random graph is $\frac{1}{2}n(n-1)p$.

   (b) the expected number of triangles T (triples of points each pair of which is joined by an edge) is $\frac{1}{6}n(n-1)(n-2)p^3$.

8. Imagine you are playing the following game: a fair die is thrown n independent times with outcomes $X_1, X_2, \ldots, X_n$. After each throw you are allowed to stop the game in which case you get payed the amount shown on the die X_k. Your goal of course is to maximize the payoff (at least in an average sense) by choosing an optimal stopping strategy. Based on the information available at that time, any stopping rule must tell you whether to stop or continue.

   (a) Based on your gut feeling, how much would you pay to participate in this game with $n = 4$?

   (b) Find an optimal stopping strategy first for $n = 2$ and then for $n = 3, 4$. The quantity you should optimize is the expected payoff.

   (c) Let $n = 4$. Compute the fair prize for participating in this game.