

Math 55 Worksheet 6

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OH: Tues 10-12pm, Evans 1047

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1 Recall

1. What is the definition of $a \mid b$?
2. What is the definition of $a \equiv b \pmod{m}$?
3. What is the definition of $\gcd(a, b)$?
4. What is the definition of *relatively prime*?
5. State Bezout's Theorem.

2 Problems

1. True or False: If $a \mid (mn)$ then either $a \mid m$ or $a \mid n$.
2. What is the base 5 expansion of 154? What is the base 3 expansion of 172?
3. Evaluate the following:
 - (a) $-23 \pmod{4}$.
 - (b) $(32^3 \pmod{13})^2 \pmod{11}$
 - (c) $(7^3 \pmod{23})^2 \pmod{31}$
 - (d) $(21^2 \pmod{15})^3 \pmod{22}$
4. Calculate $\gcd(224, 126)$. Write the gcd as a linear combination of 224 and 126.
5. Calculate $3^{2003} \pmod{99}$.
6. Ask questions!

3 Extra

7. Let $a, b, c \in \mathbb{Z}$. Prove that if $a \mid b$ and $b \mid c$, then $a \mid c$.
8. Let $a, b, c, m \in \mathbb{Z}$. Prove that if $a \equiv b \pmod{m}$, then $ac \equiv bc \pmod{m}$. Is it true that if $ac \equiv bc \pmod{m}$, then $a \equiv b \pmod{m}$?
9. Let $a, b, c \in \mathbb{Z}$ such that $a^2 + b^2 = c^2$. Prove that at least one of a, b is even. (Hint: Look at $c^2 \pmod{4}$).
10. Let A be a palindromic positive integer with an even number of digits. For example, A could be 403,304. Prove that A is divisible by 11. (Hint: write A out in base 10 expansion).