Math 55 Worksheet 3

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OH: Tues 10-12pm, Evans 1047

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1 Problems

1. Negate the following statement and rewrite it so that all negation symbols immediately precede predicates (i.e. no negation symbols before quantifiers $\forall, \exists$).

$$\forall x \exists y (P(x,y) \wedge \exists z R(x,y,z))$$

2. The symbol $\exists! x P(x)$ stands for "there exists one and only one x such that P(x) is true" and is often pronounced "there is a unique x..." Show that $\exists! x P(x)$ can be rewritten using just regular quantifiers.

3. Take the following premises for granted:

   (a) The rebellion will fail if Palpatine is not defeated.
   (b) Palpatine can be defeated only if Luke joins the dark side or Darth Vader has a change of heart.
   (c) The rebellion did not fail.
   (d) Luke did not join the dark side.

   Find someone next to you and convince them that Darth Vader had a change of heart.

4. Prove that if $m+n$ and $n+p$ are even integers, where m, n, p are integers, then $m+p$ is even.

5. Let $a, b \in \mathbb{Z}$ and suppose $a^2 + b^2 = 4$. Prove that both a and b are even.