Math 55 Worksheet 2

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OH: Tues 10-12pm, Evans 1047

September 4, 2019

Instructions

- Introduce yourselves! Despite popular belief, math is in fact a team sport!
- Try to work out the problems as a group, but feel free to flag me down if you run into a logical wall.

Logic Symbols and Propositional Equivalence

1. Let c ="It is cloudy," r ="It is raining", s ="It is sunny", w ="The ground is wet", n = "It is night". Translate each of the following either into English or into formal logical statements, as appropriate.
   (a) $c \to \neg s$
   (b) $\neg s \to c \vee n$
   (c) $r \to w \wedge c$
   (d) It is raining whenever the ground is wet.
   (e) If it's sunny, then it's not raining.
   (f) It only rains at night.
2. Without using truth tables, show that $\neg p \to (q \to r)$ and $q \to (p \vee r)$ are logically equivalent.
3. (a) Find a compound proposition that has the following truth table:

p	q	r	??
T	T	T	T
T	T	F	F
T	F	T	F
T	F	F	T
F	T	T	F
F	T	F	T
F	F	T	F
F	F	F	F

   (b) Explain how you could generalize your procedure to any number of variables and any truth table. This is what we mean when we say $\neg, \wedge, \vee$ are a *complete* set of connectives.

Predicates and Quantifiers

1. Let $S(x)$ be the statement "x is a student," $L(x)$ be "x lives in Germany" $G(x)$ be "x speaks German." Translate each of the following into English or into logic symbols as appropriate. The domain is the set of all people.

   (a) $\exists x\,(L(x) \wedge S(x))$
   (b) $\forall x(L(x) \wedge \neg S(x) \rightarrow G(x))$
   (c) $\exists x(S(x) \wedge L(x) \wedge \neg G(x))$
   (d) There is a German speaking student.
   (e) Not all speakers of German live in Germany.
   (f) The only German residents who don't speak German are students.
   (g) Some students live in Germany, but some don't.

2. Determine whether each of the following pairs of sentences are equivalent. If so, explain why. If not, give an example of predicates and domains where they differ.

   (a) $\exists x(P(x) \wedge Q(x))$; $\exists x P(x) \wedge \exists x Q(x)$
   (b) $\forall x(P(x) \wedge Q(x))$; $\forall x P(x) \wedge \forall x Q(x)$
   (c) $\exists x(P(x) \rightarrow Q(x))$; $\exists x P(x) \rightarrow \exists x Q(x)$

3. Determine the truth value of each of the following statements. The domain is the set of all real numbers

   (a) $\forall x \exists y(x > y)$
   (b) $\exists x \exists y(x \geq y \wedge y \geq x)$
   (c) $\forall x \exists y(x = y^2)$
   (d) $\forall x \forall y \exists z(x > y \rightarrow x > z > y)$
   (e) $\exists x \exists y(x + y = 1 \wedge x - y = 3)$
   (f) (**hard**) $\forall \epsilon > 0\ \exists \delta > 0\ \forall x\ (|x - 3| < \delta \rightarrow |x^2 - 9| < \epsilon)$

4. The symbol $\exists! x P(x)$ stands for "there exists one and only one x such that P(x) is true" and is often pronounced "there is a unique x..." Show that $\exists! x P(x)$ can be rewritten using just regular quantifiers.

Rules of Inference

1. Take the following premises for granted:

   (a) The rebellion will fail if Palpatine is not defeated.
   (b) Palpatine can be defeated only if Luke joins the dark side or Darth Vader has a change of heart.
   (c) The rebellion did not fail.
   (d) Luke did not join the dark side.

   Prove, citing the proper rules of inference, that Darth Vader had a change of heart.