Math 55 Worksheet 15

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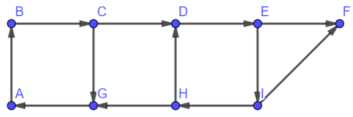
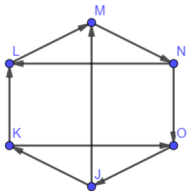
1 Problems

1. Find the strongly connected components of the graphs below.





2. Show that being bipartite is a graph invariant. Are the following graphs isomorphic?

3. How many vertices and edges are in the following graphs:

   (a) The complete graph K_n.

   (b) The cycle graph C_n.

   (c) The wheel graph W_n.

   (d) The n-cube Q_n.

4. For which values of n are the following graphs bipartite?
   (a) The complete graph K_n.
   (b) The cycle graph C_n.
   (c) The wheel graph W_n.
   (d) The n-cube Q_n.
5. Find the adjacency matrix for the following graphs.
   (a) K_n
   (b) C_n
   (c) W_n
6. Let B be the incidence matrix of an undirected graph. What is the sum of entries in a row of B? A column of B?
7. Let $A = \begin{pmatrix} 1 & 1 & 0 \\ 1 & 0 & 2 \\ 0 & 2 & 0 \end{pmatrix}$ be the adjacency matrix for an undirected graph G on three vertices v_1, v_2, v_3. Draw G. How many paths are there from v_1 to v_2 of length 1? Of length 2? Of length 3? What about paths from v_1 to v_3? Compute A^2 and A^3 and compare your answers to the $(1,2)$ and $(1,3)$ entries.
8. How many nonisomorphic connected simple graphs are there with n vertices when n is
   a) 2? b) 3? c) 4?

2 Challenges

9. A *tree* is a connected graph without cycles. Show that the following are equivalent definitions of a tree:
   (a) A maximally acyclic graph (i.e. adding any edge will result in a cycle)
   (b) A minimally connected graph (i.e. removing any edge will result in a disconnected graph).
   (c) A graph such that there exists a unique path between any two vertices.
10. Let T be a tree. Show that T is bipartite.
11. A graph is *planar* if it can be drawn without any edges intersecting (edges don't have to be straight lines). Which of the following graphs do you think are planar?
    (a) K_3? K_4? K_5? (b) $K_{2,2}$? $K_{3,3}$?