Math 55 Worksheet 11

Jeremy Meza

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1 Warm-up

1. What is the coefficient of x^6y^3 in the expansion of $(-3x+2y)^9$?
2. How many different strings can be made from the letters in MISSISSIPPI, using all the letters?

2 Problems

1. Let $n \in \mathbb{Z}_+$. Prove by a combinatorial argument that
$$\binom{2n}{2} = 2\binom{n}{2} + n^2$$
2. Let $n, k \in \mathbb{Z}_+$. Prove by a combinatorial argument that
$$k\binom{n}{k} = n\binom{n-1}{k-1}$$
3. How many ways can I give 3 distinct candies to 5 children? (some children may receive none and some may receive more than 1)
4. How many ways can I give 3 distinct candies to 5 children if no child is allowed more than 1 candy?
5. How many ways can I give 3 indistinguishable candies to 5 children if no child is allowed more than 1 candy?
6. How many ways can I give 3 indistinguishable candies to 5 children, if children are allowed more than 1 piece of candy?
7. How many ways are there to choose a dozen donuts from the 24 varieties at Kingpin Donuts?
8. How many solutions are there to the equality $x_1 + \cdots + x_k = n$, where $x_i, n \in \mathbb{N}$?

9. Count the number of 6 card hands dealt from a standard deck of 52 cards that have at least one card in every suit.

10. A *lattice path to* (a, b) is a walk starting at the origin and ending at (a, b) where at each step you are allowed to move one unit north or one unit east.

    (a) How many lattice paths are there to $(2n, 2n)$?

    (b) How many lattice paths are there to $(2n, 2n)$ that go through (n, n)?

11. How many solutions are there to the inequality $x_1 + \cdots + x_k \leq n$?

12. How many ways can n books be placed on k distinguishable shelves

    (a) if the books are indistinguishable copies of the same title?

    (b) if no two books are the same, and the positions of the books on the shelves matter?

3 Bonus

The following is called the *twelvefold way* in combinatorics. Let X, N be finite sets of size x, n, respectively. Below, "indistinguishable" means that the elements of the set are still distinct elements, but that there just might not be a difference between them. For example, indistinguishable elements could be k candies all of the same brand, whereas distinguishable elements could be k different-brand candies. Try to fill out the marked entries of the table:

	# of $f : N \to X$	# of injective $f : N \to X$	# of surjective $f : N \to X$
N distinguishable X distinguishable	(a)	(b)	
N indistinguishable X distinguishable	(c)	(d)	(e)
N distinguishable X indistinguishable		(f)	
N indistinguishable X indistinguishable		(g)	

Bonus: Try to fill out the remainder of the table.