Math 55 Worksheet 10

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1 Warm-up

Try to recall the following concepts *without* looking at your notes:
Sum Rule *Product Rule* *Inclusion-Exclusion* *Pigeonhole Principle*
r-permutation *r-combination* *binomial coefficient*
binomial theorem $\binom{n}{k}$

2 Problems

1. How many different three-letter initials can people have?

2. How many different three-letter initials with none of the letters repeated can people have?

3. How many positive integers not exceeding 100 are divisible either by 4 or by 6?

4. Find the number of 5-permutations of a set with nine elements.

5. A professor writes 40 discrete mathematics true/false questions. Of the statements in these questions, 17 are true. If the questions can be positioned in any order, how many different answer keys are possible?

6. The English alphabet contains 21 consonants and 5 vowels. How many strings of 6 English letters contain

   (a) exactly one vowel?

   (b) exactly two vowels?

   (c) at least one vowel?

   (d) at least two vowels?

7. What is the coefficient of x^4y^7 in the expansion of $(2x - y)^{11}$?

8. Let $n \in \mathbb{N}$ and suppose we have a set $S \subseteq [2n]$ of size $|S| = n + 1$. Prove that there must be two elements $x, y \in S$ that are relatively prime.

9. Let $n \in \mathbb{N}$. Prove by a combinatorial argument that

$$\binom{2n}{2} = 2\binom{n}{2} + n^2$$

10. Let $n \in \mathbb{Z}^+$. Prove by a combinatorial argument that

$$n \cdot 2^{n-1} = \sum_{k=1}^{n} \binom{n}{k} \cdot k$$

11. Let $n \in \mathbb{N}$. Prove by a combinatorial argument that

$$\sum_{k=0}^{n} 2^k \binom{n}{k} = 3^n$$