Math 55 Worksheet 1

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- 1. There are 4 persons (A, B, C and D) who want to cross a bridge in night.
 - A takes 1 minute to cross the bridge.
 - B takes 2 minutes to cross the bridge.
 - C takes 5 minutes to cross the bridge.
 - D takes 10 minutes to cross the bridge.

There is only one torch with them and the bridge cannot be crossed without the torch. There cannot be more than two persons on the bridge at any time, and when two people cross the bridge together, they must move at the slower persons pace.

What is the minimum amount of time in which they all cross the bridge? Hint: It's less than 19 min.

- 2. You are Samuel L. Jackson. You have a jug that holds 5 liters of water. Your friend Bruce Willis has a jug that holds 3 liters of water. There is a faucet in front of you. How can you measure out precisely 4 liters of water?
- 3. According to Snapchat, the fire emoji denotes a "Snapstreak" and disappears according to the rule:

If you and your friend don't both send a Snap within 24 hours, you will lose your Snapstreaks!

Within 24 hours, you send a snap and your friend Methuselah does not send a snap. Will you lose your Snapstreak according to the rule above? What about according to the rules:

- (i) If you and your friend both don't send a snap, you will lose your Snapstreaks.
- (ii) If you or your friend don't send a snap, you will lose your Snapstreaks.

- 4. Which of these statements mean the same thing?
 - (a) If it is raining then I am happy.
 - (b) If I am happy then it is raining.
 - (c) If it is not raining then I am not happy.
 - (d) If I am not happy then it is not raining.
- 5. Are there more rational numbers than integers? Are there more real numbers than rational numbers? What does "more" mean for these sets?
- 6. Consider a dart board having two regions, one worth 5 points and the other worth 3 points. What is the largest point total that cannot be obtained by throwing darts at the board?
- 7. On a remote island, you are one of 200 people, 100 of whom have blue eyes and 100 of whom have brown eyes. You cannot see the color of your own eyes (there are no reflective surfaces anywhere!), but you can see the color of everyone else's eyes. You also don't know the distribution of eye colors (for all you know, you could have purple eyes!) and no one is allowed to communicate with anyone else in any way, shape, or form.

Every night a boat appears on the island and sails away the next day. The boat is captained by a mystical guru by the name of Vivek Shende. If you go up to Vivek and correctly say the color of your own eyes, you will be allowed on the boat. If you incorrectly say the color of your own eyes, you will be cast away forever without even a volleyball for consolation.

On the first day, Vivek says to the masses: "I see someone with blue eyes." Who gets on the boat, and on what day?

- 8. How many ways are there to fill a $2 \times n$ rectangle with 2×1 domino tiles? Hint: Work it out for small n first.
- 9. Suppose 4 students take an exam and the exam papers are handed back at random for peer grading. What is the probability that no student gets their own paper back? What about for n students? What happens to this probability as $n \to \infty$?